MATH 135-08, 135-09 Calculus 1, Fall 2017
Sample Final Exam Questions

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1. **(a)** Find the equation of the line passing through the points $(1, 4)$ and $(2, 1)$.

   (b) Find the equation of the line that is perpendicular to the line in part **(a)** and passes through the midpoint of the segment between $(1, 4)$ and $(2, 1)$.

2. **(a)** State the domain and range of $f(x) = \cos^{-1}(x) + \pi$.

   (b) If $\cos\theta = -3/5$ and $\pi < \theta < 3\pi/2$, find $\cot\theta$.

   (c) Find the period and amplitude of the function $g(x) = 7\cos(x/3)$.

3. Consider the function $f(x) = \ln(x + 3) - 2$.

   (a) State the domain and range of $f(x)$.

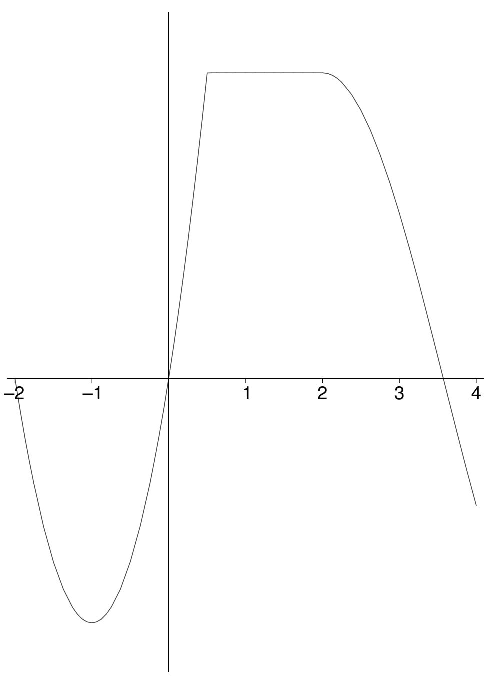
   (b) Where does $f(x)$ have a vertical asymptote?

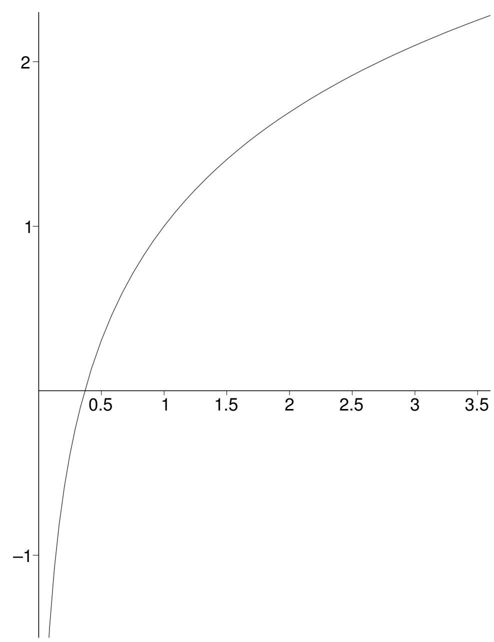
   (c) Sketch a graph of $f(x)$ and locate the exact values of the x- and y-intercepts.

   (d) Find the inverse $f^{-1}(x)$ of $f(x)$. State the domain and range of f^{-1}.

4. Find the equation of the tangent line to the curve defined by $e^{xy} + x^2 + y^2 = 10$ at the point $(0, 3)$.

5. Consider the graphs of $f(x)$ (left) and $g(x)$ (right) shown below.





   (a) At what points (if any) is $f(x)$ **NOT** differentiable?

   (b) Sketch the graphs of $f'(x)$ and $g'(x)$.

6. Evaluate each of the following limits, if they exist. Note that ∞ or $-\infty$ are acceptable answers.

   (a) $\displaystyle\lim_{t\to -2} \frac{2t^2+3t-2}{t^2-4}$

   (b) $\displaystyle\lim_{\theta\to 0} \frac{\tan(4\theta)}{\sin(5\theta)}$

   (c) $\displaystyle\lim_{x\to 0} \frac{\cos(3x)-1}{5x^2}$

   (d) $\displaystyle\lim_{x\to 1^+} \ln(\ln x)$

   (e) $\displaystyle\lim_{x\to \infty} \tan^{-1}(e^{-x}+1)$

7. Using a **LIMIT definition** of the derivative, calculate $f'(3)$ for $f(x)=\sqrt{3x}$.

8. Compute the derivative of each function. Simplify your answer as best as possible.

   (a) $f(x)=x^2e^{\tan x}$

   (b) $g(t)=\dfrac{1}{\sqrt{t^4+4t^3}}$

   (c) $h(x)=\cos(2^x)$

   (d) $y=\tan^{-1}(\ln(5x))$;

9. Suppose that $f(x)=\dfrac{x}{x^2+1}$.

   (a) What type of function is f, even, odd, or neither?

   (b) Find any vertical or horizontal asymptotes.

   (c) Calculate and simplify $f'(x)$ and $f''(x)$.

   (d) Locate and classify (min, max or neither) the critical points of f.

   (e) Locate any inflection points of f.

   (f) Using all of the information obtained above, sketch the graph of $f(x)$.

10. You wish to construct a small box by removing four congruent squares from the corners of a 3 inch by 8 inch piece of cardboard. After removing the four corners you fold up the sides to create a box with an open top. What are the dimensions of the box of largest volume you can make in this manner?

11. TRUE or FALSE. Decide whether the following statements are true or false. If true, provide an explanation. If false, correct the statement or provide a counterexample.

    (a) If a function $f(x)$ is continuous at $x=a$, then it is also differentiable at $x=a$.

    (b) The graph of $g(x)=f(-x)+3$ is obtained by shifting the graph of $f(x)$ vertically up by 3 units and reflecting it about the y-axis.

    (c) If $s(t)=e^{5t}-\ln(5t)$ gives the position of a particle at time t, then the acceleration of the particle at time $t=1$ is $26e^5$.

    (d) The two curves $y=e^x$ and $y=e^{-x}$ are perpendicular at their point of intersection. *Hint:* Two curves are perpendicular at $x=a$ if their tangent lines are perpendicular at $x=a$.

    (e) Suppose that f is a differentiable function and that $h(x)=f(\sin(x))$. If $f'(0)=3$ and $f''(0)=5$, then $h'(\pi)=-3$ and $h''(\pi)=2$.