MATH 135-08, 135-09 Calculus 1, Fall 2017

Rates of Change: Worksheet for Section 3.4

Below we consider two typical applications of the derivative from the fields of economics and physics. The key fact to remember is that dy/dx (the derivative of y with respect to x) measures the instantaneous **rate of change** of y with respect to x. So, for example, if $T(t)$ measures the temperature T (in degrees Celsius) of an object as a function of time t (in seconds), then dT/dt gives the rate of change of the temperature of the object. The units of dT/dt are $^{\circ}C/$sec. If $dT/dt > 0$, then the object is warming; if $dT/dt < 0$, then the object is cooling.

Economics: Let $C(x)$ be the cost of producing a quantity x of some item. For example, $C(25) = \$3,000$ means it costs $\$3,000$ to produce 25 of the particular item. The derivative $C'(x)$ is called the **marginal cost**. It tells us approximately how much it costs to produce the next item, the $(x+1)$st item. Similarly, if $P(x)$ is the profit made form selling x items, then $P'(x)$ is called the **marginal profit**, and if $R(x)$ is the revenue made form selling x items, then $R'(x)$ is called the **marginal revenue**.

Example 1: Suppose $C(x) = 8000 - 10x + x^2 + 0.01x^3$ represents the cost of producing x computers.

a) Find the marginal cost function.

b) Find $C'(10)$ and explain its meaning. What are the units of $C'(10)$?

c) Find the actual cost of producing the 11th computer. Compare your answer with $C'(10)$.

Physics: If $s(t)$ is the position of a moving object (or particle on a line) as a function of time t, then $s'(t) = v(t)$ is the instantaneous **velocity** and $s''(t) = v'(t) = a(t)$ is the **acceleration**. The speed of the object is defined to be $|s'(t)| = |v(t)|$, which is always positive.

Example 2: Suppose a particle moves according to the equation $s(t) = t^3 - 12t^2 + 36t$ for $t \geq 0$, where s, the position, is measured in meters and t, the time, is measured in seconds. Think of the particle moving along a number line, with s indicating the position on the line.

a) Compute the velocity and acceleration of the particle at time t.

b) When is the particle at rest?

c) When is the particle moving to the right? to the left?

d) Find the total distance traveled by the particle in the first 6 seconds.