MATH 135 Calculus 1, Fall 2015

Worksheet for Section 3.4: Rates of Change

Below we consider three typical applications of the derivative in the fields of economics, physics, and biology. The key fact to remember is that dy/dx (the derivative of y with respect to x) measures the instantaneous **rate of change** of y with respect to x.

Economics: Let $C(x)$ be the cost of producing a quantity x of some item. For example, $C(25) = \$3,000$ means it costs $\$3,000$ to produce 25 of the particular item. The derivative $C'(x)$ is called the **marginal cost**. It tells us approximately how much it costs to produce the next item, the $(x+1)$st item. Similarly, if $P(x)$ is the profit made form selling x items, then $P'(x)$ is called the **marginal profit**, and if $R(x)$ is the revenue made form selling x items, then $R'(x)$ is called the **marginal revenue**.

Example 1: Suppose $C(x) = 8000 - 10x + x^2 + 0.01x^3$ represents the cost of producing x computers.

a) Find the marginal cost function.

b) Find $C'(10)$ and explain its meaning. What are the units of $C'(10)$?

c) Find the actual cost of producing the 11th computer. Compare your answer with $C'(10)$.

Physics: If $s(t)$ is the position of a moving object (or particle on a line) as a function of time t, then $s'(t) = v(t)$ is the instantaneous **velocity** and $s''(t) = v'(t) = a(t)$ is the **acceleration**. The speed of the object is defined to be $|s'(t)| = |v(t)|$, which is always positive.

Example 2: Suppose a particle moves according to the equation $s(t) = t^3 - 12t^2 + 36t$ for $t \geq 0$, where s, the position, is measured in meters and t, the time, is measured in seconds. Think of the particle moving along a number line, with s indicating the position on the line.

a) Compute the velocity and acceleration of the particle at time t.

b) When is the particle at rest?

c) When is the particle moving to the right? to the left?

d) Find the total distance traveled by the particle in the first 6 seconds.

e) When is the particle speeding up? slowing down?

Biology: If $P(t)$ is the population of a given species (people, animals, bacteria, etc.) as a function of time t, then $P'(t)$ is the instantaneous **growth rate** of the population. Thus, if $P'(t) > 0$, the population is increasing at time t and if $P'(t) < 0$, the population is decreasing at time t. Strictly speaking, P is usually a discontinuous step function (set of data points), so we interpolate the values in between to create a smooth approximating curve that is differentiable.

Example 3: The population of a species of rabbits in a town is modeled by

$$P(t) = \frac{5e^{4t}}{4 + e^{4t}},$$

where t is in years and P is in thousands.

a) Show that the population is always increasing in size.

b) What is the long-term fate of the population? In other words, what is $\lim_{t\to\infty} P(t)$? *Hint:* Divide by the highest "power."

c) What is $\lim_{t\to-\infty} P(t)$?

d) Using parts **a)**, **b)** and **c)**, sketch the graph of $P(t)$.

e) At what time is the rabbit population growing the fastest? In other words, when does P' have a maximum? How fast is the population growing at this time?