MATH 135 Calculus 1
Sample Final Exam Questions

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1. **(a)** Find the equation of the **linear** function passing through the points $(1, 4)$ and $(2, 1)$.

   (b) Find the equation of the **exponential** function passing through the points $(1, 4)$ and $(2, 1)$.

2. Find the equation of the tangent line to the curve defined by $e^{xy} + x^2 + y^2 = 10$ at the point $(0, 3)$.

3. Suppose that $D(t)$ is the depth (in inches) of snow on your lawn t days after December 1st. Provide an interpretation in words (short paragraph) of the equations $D'(23) = 2$ and $D'(24) = -3$.

4. Compute the derivative of each function. Simplify your answer as best as possible.

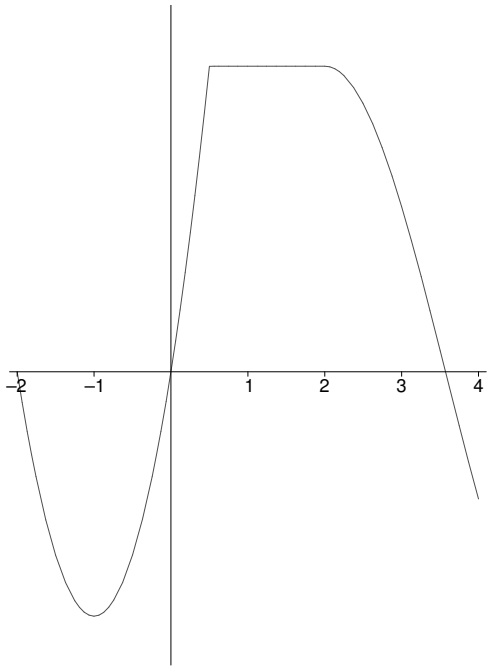
   (a) $f(x) = x^2 e^{\tan x}$

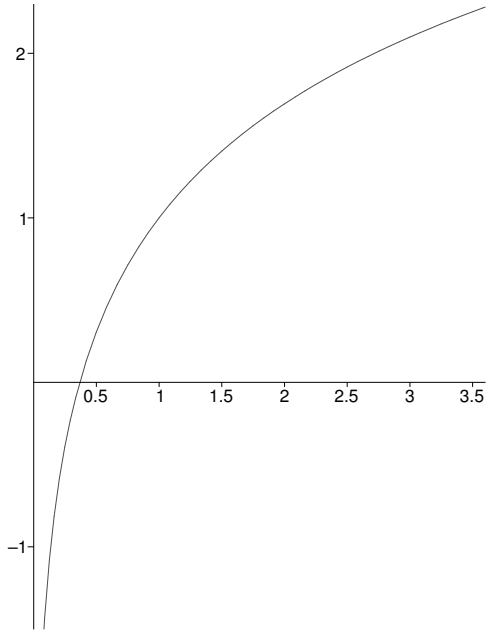
   (b) $g(t) = \dfrac{1}{\sqrt{t^4 + 4t^3}}$

   (c) $h(x) = \cos(2^x)$

   (d) $y = \tan^{-1}(\ln(5x))$;

5. Consider the graphs of $f(x)$ (left) and $g(x)$ (right) shown below.





   (a) At what points (if any) is $f(x)$ **NOT** differentiable?

   (b) Sketch the graphs of $f'(x)$ and $g'(x)$.

6. Using a **LIMIT definition** of the derivative, calculate $f'(-3)$ for $f(x) = \dfrac{4}{x}$.

7. Evaluate each of the following limits, if they exist. Note that ∞ or $-\infty$ are acceptable answers.

   (a) $\lim\limits_{t \to -2} \dfrac{2t^2 + 3t - 2}{t^2 - 4}$

   (b) $\lim\limits_{x \to 0} \dfrac{\cos(3x) - 1}{5x^2}$

   (c) $\lim\limits_{x \to 1^+} \ln(\ln x)$

   (d) $\lim\limits_{x \to \infty} \tan^{-1}(e^{-x} + 1)$

8. Suppose that $f(x) = \dfrac{x}{x^2 + 1}$.

   (a) Find any vertical or horizontal asymptotes.

   (b) Calculate and simplify $f'(x)$ and $f''(x)$.

   (c) Locate and classify (min, max or neither) the critical points of f.

   (d) Locate the inflection points of f.

   (e) Using all of the information obtained above, sketch the graph of $f(x)$.

9. You wish to construct a small box by removing four congruent squares from the corners of a 3 inch by 8 inch piece of cardboard. After removing the four corners you fold up the sides to create a box with an open top. What are the dimensions of the box of largest volume you can make in this manner?

10. A manufacturer has been selling 1000 television sets a week at \$450 each. A market survey indicates that for each \$10 reduction in the price of a television, the number of sets sold will increase by 100 per week.

    (a) Find the demand function, assuming it is linear.

    (b) What should the price of a television set be in order for the company to maximize its revenue?

    (c) If its weekly cost function is $C(x) = 68{,}000 + 150x$, what should the price of a television set be in order for the company to maximize its profit?

11. TRUE or FALSE. Decide whether the following statements are true or false. If true, provide an explanation. If false, correct the statement or provide a counterexample.

    (a) The parametric equations $x(t) = \sin(4t), y(t) = \cos(4t)$, $0 \leq t \leq \pi$ trace out the unit circle twice around in the clockwise direction.

    (b) If a function $f(x)$ is continuous at $x = a$, then it is also differentiable at $x = a$.

    (c) The graph of $g(x) = f(-x) + 3$ is obtained by shifting the graph of $f(x)$ vertically up by 3 units and reflecting it about the y-axis.

    (d) If $s(t) = e^{5t} - \ln(5t)$ gives the position of a particle at time t, then the acceleration of the particle at time $t = 1$ is $26e^5$.

    (e) If $f'(x) = \frac{6}{\sqrt[3]{x}} + 12x^5$ and $f(1) = 5$, then $f(x) = 9x^{2/3} + 2x^6 - 6$.