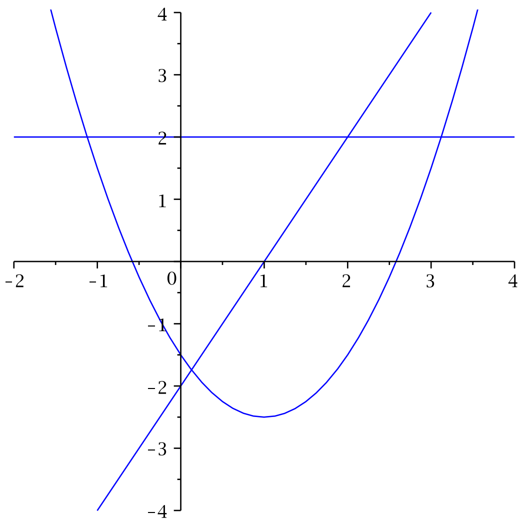
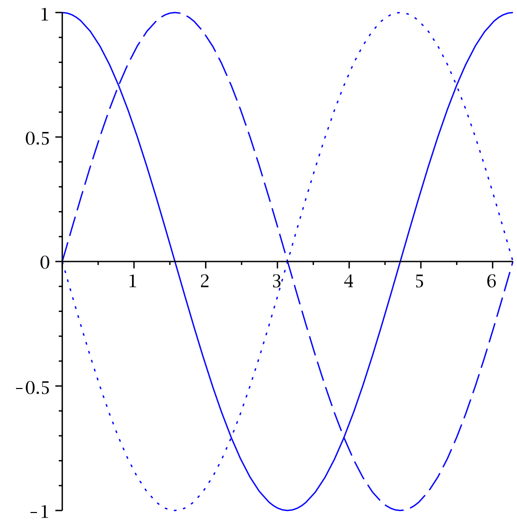
MATH 135 Quiz #5 Solutions

October 11, 2013 Prof. G. Roberts

1. Each figure below shows the graph of three functions **a, b** and **c** representing the original function f, the derivative function f' and the second derivative function f''. Identify which curve is which. (5 pts.)





a. f
b. f''
c. f'

a. f'
b. f
c. f''

Solution: For the figure on the left, the parabola **a** is decreasing (negative derivative) and then increasing (positive derivative) with a horizontal tangent line (derivative zero) at $x = 1$. The graph of **c** is negative for $x < 1$, positive for $x > 1$ and zero at $x = 1$, so **c** is the graph of the derivative of **a**. Since **c** is just a line with constant positive slope, the graph of its derivative should be a horizontal line above the x-axis. Therefore, **b** is the graph of the derivative of **c**.

For the figure on the right, the dashed curve **b** is increasing (positive derivative) and then decreasing (negative derivative) and then increasing again, with horizontal tangent lines (derivative zero) at approximately $x = 1.5$ and $x = 4.7$. The graph of **a** is positive for $x < 1.5$, negative for $1.5 < x < 4.7$ and positive for $x > 4.7$, so **a** is the graph of the derivative of **b**. The solid curve **a** is decreasing for $x < 3.1$ and increasing for $x > 3.1$, with horizontal tangent lines at $x = 0$, $x = 3.1$ and $x = 6.2$. Since **c** is negative for $x < 3.1$, positive for $x > 3.1$ and zero at $x = 0$, $x = 3.1$ and $x = 6.2$, **c** is the graph of the derivative of **a**.

2. Using one of the limit definitions of the derivative, find $f'(2)$ for $f(x) = \sqrt{6 - x}$. (5 pts.)

   The key to this problem is to set up the difference quotient correctly, to distribute the minus sign, and to multiply top and bottom by the conjugate. The rest is just algebra and canceling the h on top and bottom, before simply plugging in $h = 0$ to arrive at $f'(2) = -1/4$.

We have

$$\begin{aligned}
f'(2) &= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{6-(2+h)}-\sqrt{4}}{h} \qquad \text{(now distribute that minus sign!)} \\
&= \lim_{h\to 0} \frac{\sqrt{4-h}-2}{h} \\
&= \lim_{h\to 0} \frac{(\sqrt{4-h}-2)}{h} \cdot \frac{(\sqrt{4-h}+2)}{(\sqrt{4-h}+2)} \qquad \text{(multiply top and bottom by the conjugate)} \\
&= \lim_{h\to 0} \frac{4-h-4}{h(\sqrt{4-h}+2)} \\
&= \lim_{h\to 0} \frac{-h}{h(\sqrt{4-h}+2)} \\
&= \lim_{h\to 0} \frac{-1}{\sqrt{4-h}+2} \qquad \text{(once the } h\text{'s cancel, you can plug in } h=0\text{)} \\
&= -\frac{1}{4}.
\end{aligned}$$