MATH 134 Calculus 2 with FUNdamentals

Practice Exam #3

1. Quickies/Multiple Choice:

(a) Given the infinite series $\sum_{n=1}^{\infty} a_n$, suppose that $\lim_{n \to \infty} a_n = 0$. What can we definitively

conclude about the series?

- (i) The series converges.
- (ii) The series converges and the sum is 0.
- (iii) The series diverges.
- (iv) The series may or may not converge; the test is inconclusive.
- (b) Determine whether the sequence below converges or diverges. If it converges, state the limit.

$$a_n = \tan^{-1}\left(\frac{n^2}{2n+3}\right)$$

(c) Find the sum of the infinite series $16 - 4 + 1 - \frac{1}{4} + \frac{1}{16} - + \cdots$.

2. Compound Interest and Present Value: (round answers to the nearest cent)

- (a) Suppose that \$40,000 is invested in an account paying interest at an annual rate of 6% with the interest compounded **monthly.** How much money will be in the account after 10 years?
- (b) Congratulations, you just won a million dollars in the lottery! However, you do not get all of your money now; you will receive four yearly payments of \$250,000 beginning immediately. Assuming an interest rate of 4%, what is the present value of your prize? How much do you "lose" by not receiving the full prize today?
- 3. **Improper Integrals:** Determine whether each improper integral converges or diverges. If the integral converges, give the exact value of the integral.

(a)
$$\int_1^\infty e^{-3x} dx$$

(b)
$$\int_{2}^{5} \frac{1}{2x-4} dx$$

4. Probability: Consider the piecewise function

$$p(x) = \begin{cases} 0 & \text{if } x < 0\\ \frac{C}{(x+2)^3} & \text{if } x \ge 0 \end{cases}$$

- (a) Find the value of C that makes p(x) a probability density function.
- (b) Using your value of C from part (a), find $P(0 \le x \le 2)$.
- (c) Using your value of C from part (a), find $P(x \ge 2)$.
- 5. Infinite Series: Determine whether the given infinite series converges or diverges using any of the tests discussed in class. You must state the test used and provide valid reasons to receive full credit.

(a)
$$\sum_{n=1}^{\infty} \frac{n^3}{n^4 + 2}$$

(b)
$$\sum_{n=2}^{\infty} \frac{1}{(\sqrt{n})^3}$$

(c)
$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$$

(d)
$$\sum_{n=1}^{\infty} \left(-\frac{2}{5}\right)^n$$

6. Calculus Potpourri:

- (a) Find the arc length of $y = 2x^{3/2}$ from x = 0 to x = 1/3.
- (b) Suppose that the demand curve for a certain commodity is given by p(x) = 60 4x and the supply curve is given by $s(x) = \frac{1}{10}x^2 + 10$. Find the equilibrium price \overline{p} and then compute the consumer and producer surplus at the equilibrium price.
- (c) Find the total area of the infinitely many circles on the interval [0, 1] in the figure below.

