# MATH 134 Calculus 2 with FUNdamentals

## Practice Exam #1

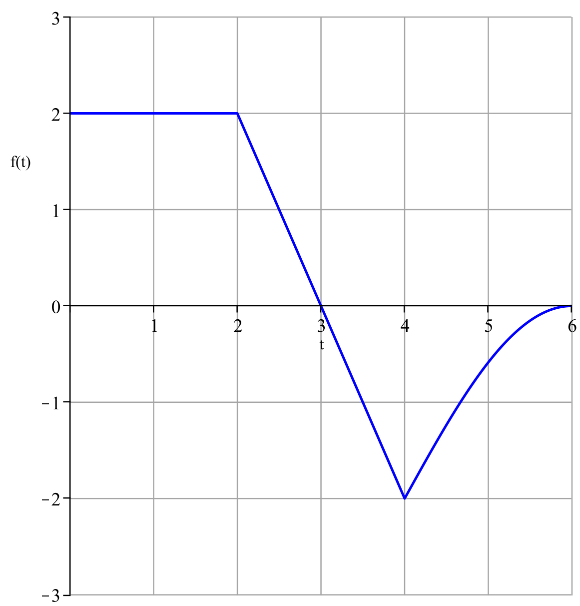
1. Let $g(x) = \ln x$ over the interval $1 \le x \le 3$.

   **(a)** Approximate the area (to three decimal places) under the graph of $g(x) = \ln x$ from $1 \le x \le 3$ by using **four** equal subintervals and right endpoints (i.e., calculate the right-hand sum $R_4$).

   **(b)** Sketch a graph of $g(x)$ over $[1, 3]$ and draw the four rectangles used to compute $R_4$. Based on your figure, is your estimate in part **(a)** an underestimate, an overestimate, or can this not be determined?

   **(c)** Approximate the area (to three decimal places) under the graph of $g(x) = \ln x$ from $1 \le x \le 3$ by using **four** equal subintervals and midpoints (i.e., calculate the midpoint sum $M_4$).

2. Define $A(x) = \displaystyle\int_0^x f(t)\,dt$ for $0 \le x \le 6$, where the graph of $f(t)$ is shown below.




   **(a)** Find $A(0)$ and $A(3)$.

   **(b)** Find $A'(2), A'(3), A''(2)$, and $A''(3)$, if they exist.

   **(c)** On what interval(s) is $A(x)$ increasing?

   **(d)** On what interval(s) is $A(x)$ concave up?

3. Evaluate each of the following integrals, giving the **exact** answer (no decimals) for parts **(e)** and **(f)**.

   **(a)** $\displaystyle\int 10x^4 + \sqrt{x} - \pi \, dx$

   **(b)** $\displaystyle\int 3^x + \sin(4x) - \frac{2}{x} \, dx$

   **(c)** $\displaystyle\int \frac{t^3 + t}{\sqrt{t^4 + 2t^2 + 7}} \, dt$

   **(d)** $\displaystyle\int \frac{x^2}{x^6 + 1} \, dx$ **Hint:** Let $u = x^3$.

   **(e)** $\displaystyle\int_{-\pi/4}^{\pi/4} \cos(2\theta)\, e^{\sin(2\theta)} \, d\theta$

   **(f)** $\displaystyle\int_0^1 \frac{(\tan^{-1} x)^3}{1 + x^2} \, dx$

4. Suppose that the acceleration of a particle traveling along a line is given by

$$a(t) = e^{3t} - 4t.$$

   If the initial velocity is $v(0) = 4$ and the initial position is $s(0) = 1$, find the position function $s(t)$.

5. Evaluate $\displaystyle\int_0^{5/4} \frac{1}{\sqrt{25 - 4x^2}} \, dx$ using the substitution $u = \dfrac{2}{5}x$. Give the **exact** answer (no decimals).

6. **Calculus Potpourri:**

   **(a)** Suppose that $\displaystyle\int_{-3}^{0} f(x)\, dx = 5$ and $\displaystyle\int_0^6 f(x)\, dx = 3$, and that $f(x)$ is an **odd** continuous function. Find the value of $\displaystyle\int_3^6 4f(x)\, dx$.

   **(b)** Find the value of $\displaystyle\int_{-3}^{3} 4\sqrt{9 - x^2}\, dx$ by interpreting the definite integral in terms of area.

   **(c)** A particle travels in a straight line with velocity $v(t) = 3t - 3$ m/s. Find the total distance traveled by the particle over the time interval $[0, 4]$.

   **(d)** Find and simplify $\displaystyle\frac{d}{dx}\left(\int_{\sqrt{x}}^{2020} \tan(t^2 + 1)\, dt\right)$.