MATH 133 Calculus 1 with FUNdamentals

Sample Final Exam Questions

Prof. G. Roberts

1. **(a)** Find the equation of the line passing through the points $(1, 4)$ and $(2, 1)$.

   (b) Find the equation of the line that is perpendicular to the line in part **(a)** and passes through the midpoint of the segment between $(1, 4)$ and $(2, 1)$.

2. **(a)** State the domain and range of $f(x) = \cos^{-1}(x)$.

   (b) If $\cos\theta = -3/5$ and $\pi < \theta < 3\pi/2$, find $\cot\theta$ and $\csc\theta$.

   (c) Find the period and amplitude of the function $g(x) = 7\cos(x/3)$.

3. Consider the function $f(x) = \ln(x + 3) - 2$.

   (a) State the domain and range of $f(x)$.

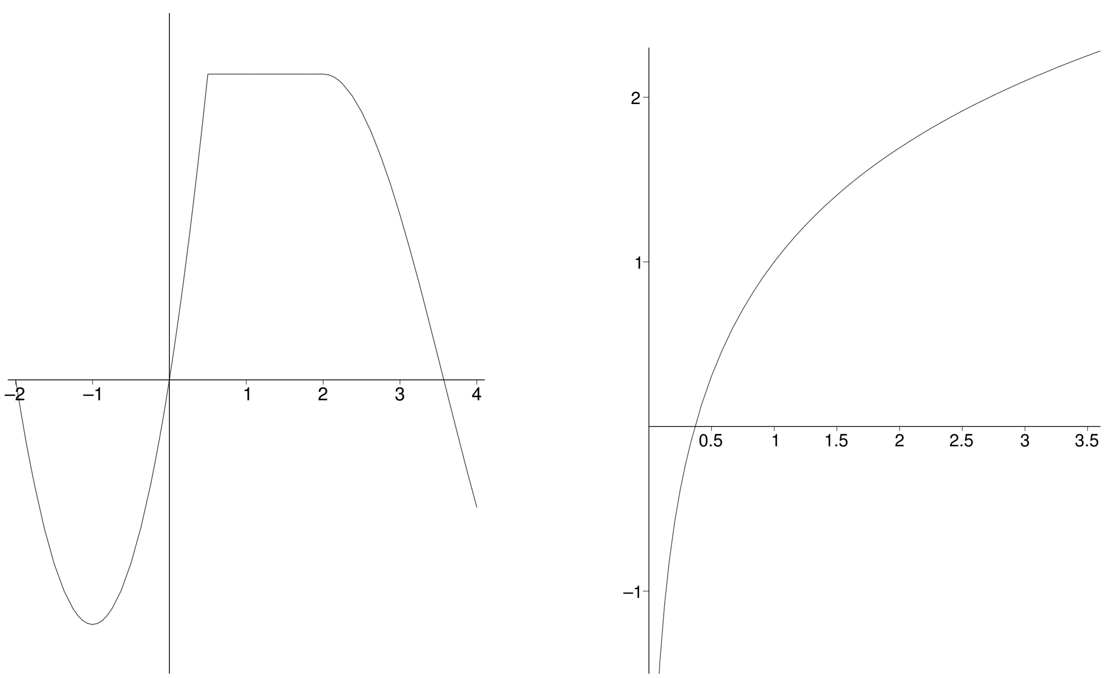
   (b) Where does $f(x)$ have a vertical asymptote?

   (c) Sketch a graph of $f(x)$ and locate the exact values of the x-intercept.

   (d) Find the inverse $f^{-1}(x)$ of $f(x)$. State the domain and range of f^{-1}.

4. Find the equation of the tangent line to the curve defined by $\sin(y^2)+x^2y+x^2+3\tan(x-3) = 9$ at the point $(3, 0)$.

5. Consider the graphs of $f(x)$ (left) and $g(x)$ (right) shown below.



   (a) At what points (if any) is $f(x)$ **NOT** differentiable?

   (b) Sketch the graphs of $f'(x)$ and $g'(x)$.

6. Evaluate each of the following limits, if they exist. Note that ∞ or $-\infty$ are acceptable answers.

   (a) $\lim_{x \to \pi} (x \sin(3x) + \cos(3x))^{21}$

   (b) $\lim_{t \to -2} \dfrac{2t^2 + 3t - 2}{t^2 - 4}$

   (c) $\lim_{\theta \to 0} \dfrac{\tan(4\theta)}{\sin(5\theta)}$

   (d) $\lim_{x \to 0} \dfrac{2 - 2\cos(5x)}{5x^2}$

   (e) $\lim_{x \to \infty} \tan^{-1}(e^{-x} + 1)$

7. Using a **LIMIT definition** of the derivative, calculate $f'(3)$ for $f(x) = \sqrt{3x}$.

8. Compute the derivative of each function. Simplify your answer as best as possible.

   (a) $f(x) = x^2 e^{\sin^{-1} x}$

   (b) $g(t) = \dfrac{1}{\sqrt{t^4 + 4t^3}}$

   (c) $h(x) = \cos(2^x)$

   (d) $y = \ln(\ln(5x))$;

9. Suppose that $f(x) = \dfrac{x}{x^2 + 1}$.

   (a) Find any vertical or horizontal asymptotes of f.

   (b) Locate and classify (min, max, or neither) the critical points of f.

10. Let $f(x) = 4 - x^2$.

   (a) Approximate the area under the graph of $f(x)$ from $x = 0$ to $x = 2$ by computing the Left-hand sum L_4. Draw a graph showing L_4 and determine whether your approximation is an under- or over-estimate.

   (b) Approximate the area under the graph of $f(x)$ from $x = 0$ to $x = 2$ by computing the Midpoint sum M_4.

   (c) Use the FTC to compute the actual area under the graph of $f(x)$ from $x = 0$ to $x = 2$.

11. Compute each of the following definite integrals.

   (a) $\displaystyle\int_0^1 2x^3 + e^{2x}\, dx$

   (b) $\displaystyle\int_0^{\pi/6} \cos(3x)\, dx$

   (c) $\displaystyle\int_{-4}^4 4\sqrt{16 - t^2}\, dt$

12. TRUE or FALSE. Decide whether the following statements are true or false. If true, provide an explanation. If false, correct the statement or provide a counterexample.

(a) If a function $f(x)$ is continuous at $x = a$, then it is also differentiable at $x = a$.

(b) The graph of $g(x) = f(-x) + 3$ is obtained by shifting the graph of $f(x)$ vertically up by 3 units and reflecting it about the y-axis.

(c) Suppose that f is a differentiable function and that $h(x) = f(\sin(x))$. If $f'(0) = 3$ and $f'(\pi) = 4$, then $h'(\pi) = -12$.

(d) Suppose that $\int_{-3}^{0} f(x)\,dx = 5$ and $\int_{0}^{6} f(x)\,dx = 3$, and that $f(x)$ is an **odd** continuous function. Then $\int_{3}^{6} f(x)\,dx = 2$.