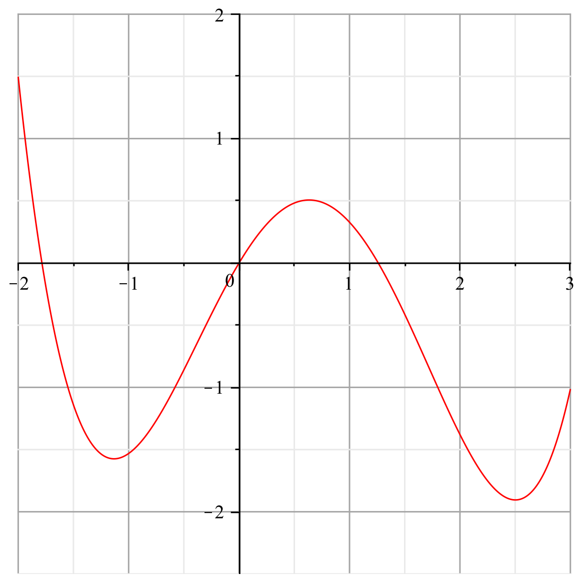
MATH 133-02 Quiz #5 Solutions

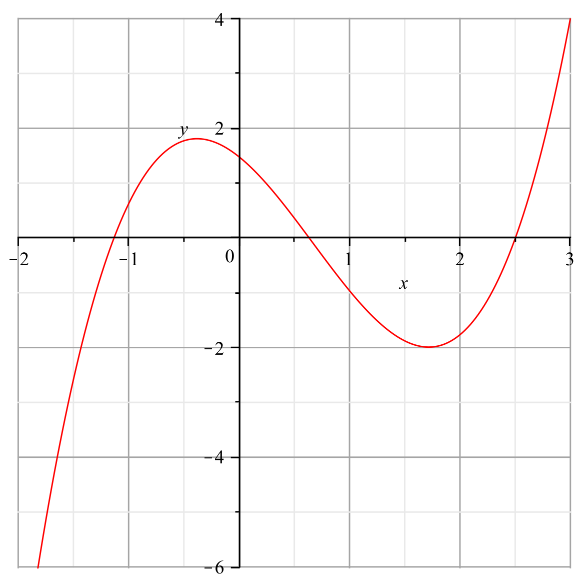
October 10, 2013   Prof. G. Roberts

1. The graph of the function $g(x)$ is shown below.



Sketch the graph of $g'(x)$ on the axes provided. (5 pts.)

Answer:



2. Using one of the limit definitions of the derivative, find $f'(2)$ for $f(x) = \sqrt{6-x}$. (5 pts.)

Answer: The key to this problem is to set up the difference quotient correctly, to distribute the minus sign, and to multiply the top and bottom of the fraction by the conjugate. The rest is just algebra and canceling the h on top and bottom, before simply plugging in $h = 0$ to arrive at $f'(2) = -1/4$.

We have

$$\begin{aligned}
f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{6-(2+h)} - \sqrt{4}}{h} && \text{(now distribute that minus sign!)} \\
&= \lim_{h\to 0} \frac{\sqrt{4-h} - 2}{h} \\
&= \lim_{h\to 0} \frac{(\sqrt{4-h} - 2)}{h} \cdot \frac{(\sqrt{4-h} + 2)}{(\sqrt{4-h} + 2)} && \text{(multiply top and bottom by the conjugate)} \\
&= \lim_{h\to 0} \frac{4-h-4}{h(\sqrt{4-h} + 2)} \\
&= \lim_{h\to 0} \frac{-h}{h(\sqrt{4-h} + 2)} \\
&= \lim_{h\to 0} \frac{-1}{\sqrt{4-h} + 2} && \text{(once the } h\text{'s cancel, you can plug in } h = 0) \\
&= -\frac{1}{4}.
\end{aligned}$$