MATH 133 Calculus 1 with FUNdamentals, Fall 2013

Worksheet on Exponential Functions (Section 1.5)

Some reminders: The general form of an exponential function is $f(x) = c \cdot a^x$ where a is called the **base** (c is an arbitrary constant). The base a is always assumed to be positive, that is, $a > 0$. If $a > 1$, we have an increasing function (**exponential growth**) that is concave up. If $a < 1$, we have a decreasing function (**exponential decay**) that is also concave up. The key feature of an exponential function is that the **ratio** between successive function-values is constant. More specifically, $f(x + 1)/f(x) = a$ is true for any x.

Examples: The functions $f(x) = 2^x, g(t) = -17 \cdot 2^t, h(x) = 400 \cdot (1/2)^x$, $q(x) = 15e^x$ and $P(t) = 3^{-t/2}$ are all examples of exponential functions. Note that

$$3^{-t/2} = (3^{-1/2})^t$$

by rules of exponents, so the function $P(t) = 3^{-t/2}$ has the base $a = 3^{-1/2} = 1/\sqrt{3}$.

The special number e: There is a very, very important number in mathematics that is a particularly useful base. It is the irrational number $e \approx 2.718281828\ldots$, which is the precise base so that the corresponding exponential function e^x has a tangent line of slope 1 at the point $(0, 1)$. This rather strange definition leads to an important fact: the derivative of e^x is just e^x. This will make more sense later on in the course.

Population models: One of the primary real-world applications of exponential functions is their use in population models. In this case, we usually write $P(t) = P_0 \cdot a^t$ where P_0 is the initial quantity, a is the base, and t is a unit of time (e.g., years or seconds). Note that $P(0) = P_0 \cdot a^0 = P_0$ so P_0 really does equal the initial amount.

Exercises:

1. Sketch the graphs of the functions $f(x) = 2^x$ and $g(x) = (1/2)^x = 2^{-x}$ on the same set of axes. *Hint:* Notice that $g(x) = f(-x)$.

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2. Find the exponential function that passes through the points $(1, 6)$ and $(3, 54)$.

3. In 1958, the price of a bleacher seat at Fenway Park (go Red Sox!) was just \$0.75. In 2004, it had risen all the way to \$20.00. Using an exponential model of the form $P(t) = P_0 \cdot a^t$, where P is the price of a bleacher seat and t is time in years, find a formula for the price of a bleacher seat at Fenway Park. What does your model imply the price should be this year? (For comparison, it is currently \$28.00.)

4. The number of cells in a particular virus doubles every three hours. If the initial population of cells is 1500, find a formula for the number of cells after t hours. How many cells are there after 50 minutes?