

MATH 133 Calculus 1 with FUNdamentals, Fall 2013

Worksheet on the Derivative (Section 2.6)

The Derivative: The slope of the tangent line is called the *derivative*. The notation for the derivative at a particular point $x = a$ is $f'(a)$, which is read as “f prime of a.” For example, $f'(5) = -3$ means that the slope of the tangent line at $x = 5$ is equal to -3 . This means the function is decreasing at this point since the slope is negative.

There are two different but mathematically equivalent limit definitions for the derivative. YOU SHOULD MEMORIZE BOTH!

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}.$$

The first of these limit definitions is the one most commonly used because it is often the easiest. Remember that a is just a constant, the number in the domain where you want to find the slope of the tangent line.

Note: Both definitions are of the form $\frac{0}{0}$, which is an indeterminate form that can be anything. You will ALWAYS have to do some type of simplification or use a calculator to compute the derivative using the limit definition. The general steps for computing derivatives using the limit definition are:

1. Find the *difference quotient* $\frac{f(a+h) - f(a)}{h}$.
2. Simplify the difference quotient. This involves algebra such as factoring, adding fractions, multiplying by the conjugate, etc.
3. Take the limit as $h \rightarrow 0$ or as $x \rightarrow a$.

Exercises:

1. Find $f'(3)$ if $f(x) = -4x + 7$. Why do you expect to get this answer?

2. Find the slope of the tangent line to the function $g(x) = \frac{2}{x} + 1$ at $x = 2$.

3. Find the equation of the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

Hint: Multiply top and bottom by the conjugate.

4. Find $g'(-2)$ if $g(x) = x^3 - 4x$.

5. Find $P'(0)$ if $P(t) = e^t$. Why does your answer make sense given the definition of e ?

Hint: You will need a calculator to do the limit.