Section 4.4, Exercise 1

Key points for essay:

Pythagorean tuning:

Strengths: uses only octave $2:1$ ratio and perfect fifth $3:2$ ratio to generate all ratios in the major scale. This is nice because they are the basic building blocks of harmony. All ratios use only powers of 2 and 3, which agrees with the Pythagorean philosophy that nature is best understood with small-numbered ratios.

Weaknesses: Two half steps do not equal one whole step, leading to the Pythagorean comma. Hard to change keys or play chromatic melodies. Circle of fifths does not close up, leading to enharmonic inequivalencies like $\mathrm{B}\sharp \neq \mathrm{C}$. The circle of fifths is really a "spiral of fifths."

Just Intonation:

Strengths: agrees well with the overtone series which means that intervals sound more consonant. In particular, the $4:5:6$ major chord will sound particularly resonant and pleasing because it comes from the 4th, 5th, and 6th partials in the overtone series. The rational numbers used to generate the scale are lower than those used for Pythagorean tuning and are thus easier to work with.

Weaknesses: Two half steps do not equal one whole step, and there are two different whole steps which differ by the syntonic comma. Still hard to change keys or play chromatic melodies. Some melodies will drift flat or sharp because of the inconsistencies within the scale. Circle of fifths does not close up; still have a "spiral of fifths" where $\mathrm{B}\sharp \neq \mathrm{C}$.

Equal Temperament:

Strengths: Two half steps equal one whole step. Consistent system with the same half step throughout. Easier to change keys and play chromatic melodies. Circle of fifths closes up. No comma's to worry about any more. This is the current system of tuning we use.

Weaknesses: Relies on an irrational number $2^{1/12}$ to generate the scale. No small-numbered ratios other than $2:1$. While the P5 is very close to $3:2$, the M3 of $2^{4/12}$ is noticeably sharp (about 14 cents) from the just M3 of $5:4$. This means that major chords do not sound as nice or pleasing as they do in just intonation.

Intervals:

The unison, M2, P4, P5, and octave are the same between Pythagorean tuning and just intonation, while the M3, m3, M6, M7 are different. All of the intervals in equal temperament are irrational except for unison and the $2:1$ octave. Thus, other than the octave, all intervals in equal temperament are different from both Pythagorean tuning and just intonation.