

Topics in Mathematics: Mathematics and Music

MATH 110, TuTh 2:00 - 3:15, Brooks 454, Spring 2018

Professor Gareth E. Roberts

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Office Hours: Mon. 10:30 - 12:00, Tues. 1:00 - 1:50, Wed. 1:00 - 2:00, Thurs. 9:45 - 10:45, or by appointment.

Required Text: *From Music to Mathematics: Exploring the Connections* by Gareth E. Roberts (copies are available on reserve in both the Music and Science libraries). You should also purchase the staff paper *Music Tablet*, available at the College bookstore.

Webpage: <http://mathcs.holycross.edu/~groberts/Courses/MA110/homepage.html>

Homework assignments, class handouts, a class calendar, exam materials, useful links and other important information will be posted at this site. Please bookmark it!

Syllabus: This is an interdisciplinary course exploring some of the connections between mathematics and music. We will consider both structural and aesthetic commonalities. For instance, both subjects use a specialized form of notation to communicate their ideas. Each has its own logical structure and set of axioms finely tuned over centuries of study. Students in high school geometry learn the axiomatic technique of Euclid to write their first proofs. Students in musical theory learning to compose for four voices are taught to avoid parallel fifths and octaves just as Bach did in his choral works. Mathematicians use numbers as the invariant building blocks of their theory as musicians use pitch as the common denominator of their creations. Just as the number three has the same abstract meaning to mathematicians everywhere, the concert pitch A440 used to tune modern orchestras is a global standard.

There are aesthetic, less concrete connections to be considered as well. Both fields have produced great child prodigies (for example, Mozart and Gauss). Parents play Mozart and Bach to their children to help foster their brain development and analytic skills. Many mathematicians are outstanding musicians, while many musicians, in particular composers, possess sharp mathematical minds. Scholars often speak of the “beauty” and “purity” of mathematics, although the same lofty descriptions could equally apply to music. While music has the obvious capacity to move the spirit, great mathematical discoveries and insights are often accompanied with an overwhelming sense of elation. The great mathematician Andrew Wiles wept on camera while speaking about his incredible proof of Fermat’s Last Theorem.

Finally there are the composers and musicians who, whether they are cognizant of the fact or not, use mathematical concepts in their creations. Bach frequently used mathematical operations such as translations and reflections in his fugues to create wonderfully rich counterpoint. Bell ringers in towers throughout England have been using permutations, symmetry, and group theory in change ringing (sometimes called campanology) to announce important events since the early 1600’s. The British composer Sir Peter Maxwell Davies used magic squares as an architectural framework for several of his pieces. The modern composer Xenakis used computers and probability theory to create his “stochastic music.”

The pertinent mathematical and musical concepts will be developed as the need arises. Musical topics to be considered include basic music theory (notation, rhythm, pitch, scales, intervals, circle of fifths, chords, progressions), non-standard tunings and scales, the overtone series, change

ringing, and modern mathematical composers such as Davies and Reich. The mathematical topics to be studied include graphs, trigonometry, logarithms, equivalence relations, modular arithmetic, group theory, rational and irrational numbers, symmetry, phase shifts, and magic squares.

A tentative outline of the course is given below. We will cover material from nearly every chapter in the course text. In addition, a few days have been reserved for a course lab using a simple musical instrument called a monochord.

- Introduction: course overview, highlights, intro questionnaire (1 class)
- Rhythm: time signature, geometric series, polyrhythms, least common multiple (2 classes)
- Special Class: experiencing polyrhythms with the **Silkroad Ensemble**, Tues., Feb. 6
- Concert: **Silkroad Ensemble**, Thurs., Feb. 8, 7:30 pm, Brooks Concert Hall (**required**)
- Basic Music Theory: scales, circle of fifths, keys, intervals, tonality, transposition (4 classes)
- The Science of Sound: how we hear, the ear-brain system, attributes of sound, sine waves, understanding pitch, a vibrating string, the overtone series, beats, the monochord lab (4 classes)
- Tuning and Temperament: the Pythagorean scale, just intonation, equal temperament, cents, rational and irrational numbers, alternative tuning systems (3 classes)
- Musical Group Theory: symmetry in music, group theory, symmetries of the square, examples (3 classes)
- Change (Bell) Ringing: basic rules and terminology, permutations, examples of extents, more group theory (3 classes)
- Mathematical Modern Music: Davies (magic squares), Reich (phase shifts), Xenakis (stochastic music) (2 classes)
- Final Project Work Days (last 2 classes)
- Final Project Performances (concert during the final exam period)

- Course Objectives:**
1. Investigate the multiple connections between mathematics and music.
 2. Develop skills in critical thinking and abstract reasoning.
 3. Develop an understanding of music theory and a deeper appreciation for music.
 4. Integrate your artistic and analytical skills.
 5. Have FUN while learning.

Homework: Homework will be assigned on a regular basis (approximately 7–8 assignments for the semester). Assignments will be posted on the course webpage. While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. If you use the Internet for help on homework, be sure to cite the website(s) visited. Take care to write up solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of the Departmental Policy on Academic Integrity.

Note: Late homework will receive a grade deduction.

Concert Reviews: You are required to attend two musical performances during the semester and turn in a typed, 1-2 page review of the concert. The purpose of these reviews is to enhance your musical appreciation, to support your fellow students and the arts, and to witness any possible connections to course material. A schedule of upcoming concerts is linked from the course webpage.

Note: Each review is due within two weeks of the concert.

Final Project: A major goal of the course is for you to complete a final project consisting of a musical composition and performance demonstrating some of the mathematical concepts learned in the course. A brief, typed report should accompany your composition explaining the mathematical connections and rationale in your work. You will also give a brief presentation describing the work immediately before its performance. The performances will take place during the scheduled final exam period. The last two class periods will be allocated for you to work on your project and to receive constructive criticism.

Exams: There will be 2 midterm exams given during class, as scheduled below. Please make a note of these dates and plan accordingly. Any conflicts must be legitimate and brought to my attention well before the exam is scheduled. If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to obtain approval from the Office of Disability Services (Hogan 215A, x3693).

Exam Schedule:

Exam 1	Thurs., March 1	In class
Exam 2	Thurs., April 19	In class

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. Anyone who violates the Departmental Policy on Academic Integrity will result in a 0 for that assignment or exam, and a letter describing the occurrence of academic dishonesty will be sent to your Class Dean.

Diversity and Inclusion: It is my intent that students from all diverse backgrounds and perspectives be well-served by this course, that students' learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed as a resource, strength, and benefit. Any suggestions you have pertaining to diversity and inclusion are encouraged and appreciated.

Grade: Your course grade will be based on the following breakdown:

- participation (includes in-class work, preparedness, attitude, effort) 5%
- homework assignments (including lab) 25%
- concert reviews 8%
- midterm exams 35%
- final project 27%

How to do well in this course:

- Attend class, participate, and ask questions. Be an engaged learner.
- Do your homework regularly.
- Work with your classmates.
- Ask for HELP when necessary.

May not Music be described as the Mathematic of Sense, Mathematics as the Music of reason? The soul of each the same! Thus the musician feels Mathematic, the mathematician thinks Music, — Music the dream, Mathematic the working life, — each to receive its consummation from the other.

— James Joseph Sylvester (1865)