MATH 110: Mathematics and Music
Homework Assignment #5
DUE DATE: Mon., March 19th, start of class.

You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your own work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the assignment.

> *Equal temperament is a lot easier to use than Just Intonation, but I find it lacks expressiveness. It sounds dead and lifeless to me. As soon as I began working microtonally, I felt like I moved from black and white into color. I found that certain combinations of intervals moved me in a deep physical way. Everything became clearer for me, more visceral and expressive. The trade-off is that I had to be a lot more careful with my compositions, for while I had many more interesting consonant intervals to chose from, I also had new kinds of dissonances to avoid.* Robert Rich (composer)

1. Read Chapter 1 of the course text, *Tuning and temperament: closing the spiral* by Neil Bibby. As you read, it is useful to do some of the computations in the text to further your understanding. Much of this material was covered in class.

2. What famous scientist tried to solve the musical difficulties of Just Intonation by using symmetry and linking the seven notes of the major scale to the seven colors of the spectrum?

3. Consider the Pythagorean C major scale. Derive the frequency ratios for F♯ and G♭ given in the text on pg. 19, assuming that C is the tonic (Do). For F♯, you should go up in pitch while for G♭, you should go down in pitch but return to the original octave. What is the ratio of the two fractions F♯ : G♭? This should look familiar.

4. In a brief essay, compare and contrast the three tuning systems: **Pythagorean Tuning**, **Just Intonation** and **Equal Temperament**. What are the strengths and weaknesses of each system? Which intervals are the same and which are different? What does Bibby mean by "closing the spiral?"

5. Let C' be the note an octave below middle C. Starting in the bass clef, use staff paper to write the first ten notes corresponding to the frequencies in the overtone series of C'. Give the interval between successive notes in the series.

6. Make a chart showing the frequencies (rounded to the nearest tenth, ie. one decimal place) of all the notes in the A major scale starting on A 220 Hz. Do this for **each** of the three different tuning systems. We started this in class computing the frequencies of C♯ and E.

7. Assuming that A 440 Hz is the note A above middle C, find the following frequencies in Hz (round to the nearest tenth) using the given tuning system. Be sure to explain how you arrived at each answer.

   a. F above middle C using Just Intonation

   b. F above middle C using Pythagorean Tuning

   c. Middle C using Just Intonation

   d. Middle C using Equal Temperament

8. Using Equal Temperament, what factor do you multiply a frequency by to raise the note up a tritone (diminished 5th)? What factor raises the note by a minor seventh? Be sure to simplify your answers.

   Recall: A number is **rational** if it can be expressed as the ratio of two integers $\frac{p}{q}$. A number is **irrational** if it is not rational.

9. Using a proof technique demonstrated in class, prove that $\sqrt{3}$ is irrational and prove that $2^{1/12}$ is irrational.

10. Prove that the sum of any two rational numbers is rational. Mathematically, this means that the set of rational numbers is *closed* under addition. *Hint:* How do we add two fractions together? Show that the result is still rational.

11. Prove that the sum of a rational and an irrational number is always irrational. *Hint:* Suppose this wasn't true and that the sum was rational. Why does this lead to a contradiction of the previous question?

12. Are the irrational numbers closed under addition? In other words, is it true that the sum of two irrationals is **always** irrational? Explain with a proof or a counterexample.