MATH 110: Mathematics and Music
CD #3: Musical Group Theory and Modern Music

This CD contains some of the examples of musical group theory we have studied in class. By musical group theory we mean the use of various mathematical symmetries such as translation, horizontal and vertical reflection, and rotation by 180°, which taken in appropriate subsets form a group (eg. the dihedral group of degree 4 – symmetries of the square). Some composers use these ideas in music quite intentionally to generate new thematic material out of an original idea. Bach was certainly one of the great masters of this, although it is important to note that he did so while never losing site of the musical art in his creations. Not every inversion or retrograde is exactly accurate for want of a musical flavor or style.

We also include some more modern mathematical music from the 20th century, featuring the twelve-tone method of Schoenberg and the purely mathematical creations of Xenakis. As discussed in CD #2, a rather radical shift occurs at the start of the 20th century, when many composers discarded tonality altogether seeking more freedom and flexibility in utilizing all the notes of the chromatic scale, rather than favoring one over the others. Schoenberg writes "Tonality is not an eternal law of music, but simply a means toward the achievement of musical form." Despite this assessment, it is interesting to note that Schoenberg still utilizes the same musical ideas as Bach (inversion, retrograde, transposition) to build his music out of a given tone row. Xenakis took the atonality concept even further, composing "sound blocks" based on some very precise mathematical calculations involving probability theory.

What to listen for: The music for many of the pieces on this CD was distributed in class. Try and follow along, reading the music as you listen, as this helps identify the composers intent. It is also a good way to engage your brain while you listen. Try to hear where a retrograde or an inversion takes place in the music. As with CD #2, this collection contains a wide variety of musical styles and various tonalities. Try to discern the differences between earlier and later forms of composition as well as between different forms of music (fugue, oratorio, symphony, sonata, song, etc.)

1. Guillaume de Machaut (c 1300 - 1377), *Ma Fin Est Mon Commencement* (My End is My Beginning). Track 19 from a Hyperion CD entitled Messe De Notre Dame performed by the outstanding Hilliard Ensemble. As the text indicates, this work is a retrograde cannon in music and almost in words as well. This is perhaps the first known example of a retrograde in music that was actually written down. Note also that the piece is sung with tuning using the Pythagorean Scale. This is particularly recognizable near the cadences or ends of each phrase. The singers are keeping as many perfect fifths as possible in a 3/2 ratio to produce the Pythagorean tuning using their voices.

2. Johann Sebastian Bach (1685 - 1750), *Fugue, "The Little"*, BWV 578. Track 8 from a CBS Records CD entitled Bach: Great Organ Favorites performed by E. Power Biggs. This is one of Bach's most popular fugues written for the organ. Recall that a fugue is a piece in which a main subject is presented successively in different voices (in this case different registers of the organ), sometimes coming back in inversion or retrograde or even retrograde-inversion. Meanwhile, a countersubject is often presented which complements the main subject heard at the outset. Often this countersubject is more active than the subject and it too may return in inversion or retrograde. Bach was arguably one of the greatest writers of fugues. Try and hear each of the different instances when the main subject reappears, noting that sometimes it shows up in a major key and sometimes in a minor key. The piece is written in G minor although it ends triumphantly on a major chord.

3. Johann Sebastian Bach, *Fugue No. 8 in D♯ minor* from *The Well-Tempered Clavier*, BWV 853, 1722 - 1742. Track 16, Disc 1 from a Radio-Canada CD entitled The Well-Tempered Clavier performed on harpsichord by Scott Ross. Bach wrote 48 preludes and fugues, one each for the 12 major keys and 12 minor keys, over a period of 22 years. The two volume work has been referred to as the pianist's Old Testament. Only through the use of equal temperament tuning is it possible to play several selections from this masterpiece. The music for Fugue No. 8 in D♯ minor was distributed in class on April 7th (part of the handout with Gershwin's *I Got Rhythm* on top). As you follow along with the music, notice the ways in which Bach brings the opening subject back, sometimes rhythmically altered, sometimes in inversion and sometimes both. Occasionally the subject appears in all three voices, but staggered, as in a round.

4. George Frideric Handel, *Hallelujah Chorus* from the *Messiah*, 1741. Track 18, Disc 2 from a BMG Entertainment CD entitled Messiah performed by Musica Sacra, directed by Richard Westenburg. Arguably Handel's most famous work, the *Messiah* is a grand oratorio telling the story of Jesus which Handel obsessively composed in only 24 days. The text is a compilation of verses from the Bible. Upon finishing the now famous Hallelujah Chorus, Handel exclaimed "I did think I did see all Heaven before me, and the Great God Himself!" Early on in the chorus (about 22 seconds in) we hear all the voices in unison singing "for the Lord God Omnipotent reigneth," a phrase which if one neglects duration, is in retrograde (see the top of p. 103 in the course text.) By using vertical symmetry, Handel is announcing the brilliance and balance of The Lord God (perhaps an early form of tone painting.) To emphasize this significance, the motif appears throughout this movement. Traditionally the audience stands when the Hallelujah Chorus is sung during a performance of the Messiah, often a powerfully spiritual moment for performers and audience alike.

5. Joseph Haydn, *Piano Sonata No. 41, "Minuet and Trio"* in A major, Hob. XVI:26, 1774-1775. Track 13, Disc 6 from the complete collection of Haydn's Piano Sonatas performed by John McCabe (Decca Record Company Limited, London). The opening minuet is titled "Menuet Al Rovescio" (Minuet in Reverse) and is an exact musical palindrome (retrograde) with the vertical reflection occurring about the end of measure 10. This music was distributed on January 20th and can be found on page 90 of the course text. The $\frac{3}{4}$ meter seems crucial to making the retrograde work harmonically as Haydn often emphasizes two of the three notes of the underlying chord in beats one and three of each measure. Thus, on playing such a measure backwards, the chord one hears remains unchanged. Haydn was so proud of his clever musical palindrome that he re-worked it into his Sonata No. 4 for Piano and Violin as well as his Symphony No. 47.

6. Ludwig Van Beethoven, *Symphony No. 5 in C minor*, 1st movement, Op. 67, 1807-1808. Track 1, Disc 3 from the complete collection of Beethoven's Nine Symphonies (he actually wrote 10, but the 10th was unfinished) performed by the Chicago Symphony Orchestra under the direction of Sir Georg Solti. This famous symphony took the idea of elaborating upon a simple motif to new heights. The opening "da da da dum" motif, once called "fate knocking at the door" by Beethoven himself, is transposed, inverted, elongated, shortened and continually repeated throughout the work to create the bulk of the music. The entire first minute of this symphony consists solely of this famous motif. Although space restrictions allow for only the first movement, you are encouraged to listen to the entire symphony to see how Beethoven brings the motif back unexpectedly in the final movement.

7. George Gershwin, *I Got Rhythm*, 1930. Track 6 off of a Concord Records, Inc. CD entitled Someone to Watch Over Me sung by Susannah McCorkle. This is a cute intro track discussing

the lyrics of the tune written by the composer's brother, lyricist Ira Gershwin.

8. George Gershwin, *I Got Rhythm*, 1930. Track 7 off of the CD listed above. Gershwin, who grew up in Brooklyn, NY, the son of Russian-Jewish parents, wrote this popular tune in his musical *Girl Crazy*. The music for this song was distributed in class on April 7th. The main opening melody is 4 bars with a simple retrograde at the end of bar 2 (with an important shift in the rhythm to give it a jazzy feel.) The form of the song is an AABA structure with the B section featuring an inversion as well as a transposition. Thus, this simple 32 bar song contains three of our four mathematical symmetries.

9. Arnold Schoenberg, *Suite for Piano, Minuett: Moderato - Trio* Op. 25, 1925. Track 20 on a Deutsche Grammophon CD entitled Schoenberg: The Piano Music performed by Maurizio Pollini. This is Schoenberg's second piece composed for piano during his twelve-tone period. The entire work (not just this movement) is based on the same 12 note tone row (discussed in class April 19th and 21st). The music for the Trio, which begins about 1:53 into the Minuett, was distributed in class on April 19th (see p. 130 of the course text.) The piece is atonal in the sense that there is no central key or tonic around which the music is based. In the Trio Schoenberg uses only the six tone rows P-0, P-6, I-0, I-6, R-6 and RI-6 (see the labels on the music in the text.) The number six is significant in that 6 half-steps equals half the octave (the tritone interval) and the tone row begins and ends a tritone apart. Each of the transformations above (transposition, inversion, retrograde and retrograde-inversion) all consequently begin and end on either E or B♭, making the music flow easier from one row to another.

10. Iannis Xenakis, *Pithoprakta*, 1955 - 1956. Track 4 from a Le Chant Du Monde CD entitled Xenakis: Eonta, Metastasis, Pithoprakta. A portion of this music along with the mathematical calculations used to create it was distributed in class on April 21st. The piece was dedicated to Hermann Scherchen who conducted its premiere in March 1957 in Munich. This innovative work is written for 46 strings (remarkably all playing different parts), 2 trombones, 1 xylophone and 1 wood block. The composers goal was to use probability theory to determine what notes should follow each other and what lengths they should be played, so-called "Stochastic Music," invented by Xenakis. The piece explores the conflict between continuity and discontinuity by juxtaposing continuous sounds (glissandi in the strings and trombones) with discontinuous ones (pizzicati plucking in the strings, tapping the wood of the strings with the opposite side of the bow, and the sharp, piercing sounds of the wood block.) Xenakis determines the "speed" of a given glissando (the slope found as the ratio of pitch to duration) by using a uniform distribution (sometimes called a normal or Gaussian distribution.) This has the mathematical effect of distributing the speeds equally among all players so that all pitches are freely distributed along a continuous frequency spectrum. In this way, Xenakis extends Schoenberg's main goal in 12-tone music of not favoring any one pitch over any other. Given Xenakis' experiences with the Greek Resistance and WWII, it is fairly obvious that this work is the composers musical reflection upon the most tragic of human conflicts – war.