MATH 110: Mathematics and Music
Homework Assignment #7
DUE DATE: Fri., April 7, start of class.

You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your own work.

Note: Please list the names of any students or faculty who you worked with on the assignment.

Music is the arithmetic of sounds as optics is the geometry of light.

Claude Debussy, c. 1900

1. Read Chapter 6 of the course text, *The geometry of music* by Wilfrid Hodges. Some of this material was covered in class at the start of the semester.

2. On page 99, Hodges gives five different classes of symmetries, *p1, ph, pv, p2, phv*. For each of the letters **B**, **I**, **L**, **T** and $\mathbf{\Theta}$, give the corresponding symmetry class.

3. At the start of the semester, we discussed three different types of musical symmetries used by composers: *transposition, retrograde and inversion.* Describe each of the five different classes of symmetries *p1, ph, pv, p2, phv* using these musical terms. For example, class *p1* corresponds to *transposition* because the only symmetry inherent in a transposition is the identity transformation. Note: Some classes may involve more than one musical term.

4. Give two examples (one from the text, one from Prof. Little's talk) of music using symmetry *p2*. What type of symmetry does Béla Bartók use in *Mikrokosmos, No. 141, Subject and reflection*?

5. **Closure:**

   a. Which of the following sets are closed under addition?
   (i) The integers **(ii)** The rationals **(iii)** The irrationals

   b. Which of the following sets are closed under multiplication?
   (i) The integers **(ii)** The rationals **(iii)** The irrationals

6. **Multiplication of permutations:** For this problem let

$$a = (1\,3\,5\,2\,4\,6) \quad \text{and} \quad b = (2\,4\,6\,1\,3\,5)$$

   be two permutations in S_6. Compute the following:

   a. $a * b$

   b. $b * a$

   c. a^2

   d. a^4

   e. b^{-1}

7. Finish completing the group multiplication table (Table 1) for the first lead of Plain Bob Minimus. Instead of doing the multiplications out by hand, try using some of the identities discussed in class: $\alpha^2 = e$, $\beta^4 = e$, $(\alpha\beta)^2 = (\beta\alpha)^2 = e$ and $\beta\alpha\beta = \alpha$. This should save you some time.

8. Notice that the completed Table 1 shows that the set of permutations from the first lead of Plain Bob Minimus (call it Set A) are closed under multiplication of permutations. Associativity follows since the larger group S_4 has associativity and $e =$ rounds (the identity element) is the first change in the lead. List the inverse of each of the eight elements of the first lead and conclude that Set A is indeed a subgroup of order 8 of S_4. (In other words, Set A is a group all by itself.)

9. Finish completing the group multiplication table (Table 2) for the symmetries of the square. This set is called the **Dihedral group of degree 4**, denoted D_4.

10. Notice that the completed Table 2 shows that the symmetries of the square are closed under composition. Associativity follows from the definition of composition of functions and the identity element is contained in the set of symmetries. List the inverse of each of the eight elements of D_4 and conclude that D_4 is a group of order 8.

11. What do you notice about any row or any column in each table?

12. Is either group (Set A or D_4) commutative? In other words, is it true that $a * b = b * a$ for **every** element a, b in the group?

13. Consider Table 2. In general, what is the composition of two rotations? of two reflections? of a rotation and a reflection?

14. **Bonus Question:** Compare Tables 1 and 2. Make a bold and remarkable conjecture concerning the groups given by Set A and D_4. Try and be precise as possible.