MATH 110: Mathematics and Music
Homework Assignment #6
DUE DATE: Fri., March 31, start of class.

You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your own work.

Note: Please list the names of any students or faculty who you worked with on the assignment.

> *Change ringing is a non-competitive and non-violent team activity that is highly stimulating intellectually and mildly demanding physically, and makes a beautiful sound. It develops mental and physical skills in a context of communal effort. The intense concentration required brings euphoric detachment that cleanses the mind of the day's petty demands and frustrations.* North American Guild of Change Ringers
> (http://www.nagcr.org/pamphlet.html)

1. Read Chapter 7 of the course text, *Ringing the changes: bells and mathematics* by Dermot Roaf and Arthur White. As you read, it is useful to follow the computations in the text to further your understanding. Some of this material was covered in class.

2. Based on the text, how do mathematicians use "graphs" to visualize an extent? Sketch an example for $n = 3$ bells.

3. How many different ways can 10 numbered bells be arranged so that the first bell is Bell 1 and the last bell is Bell 10, but all other bells are arbitrarily arranged?

4. Suppose that n is a positive integer. Simplify the expression $\frac{(n+2)!}{n!}$. *Hint:* Try plugging in small values of n and then look for a pattern. Alternatively, express $(n+2)!$ in terms of $n!$

5. Recall that for $n = 3$ bells, there are only two allowable "moves" (permutations) between changes, interchanging the inner bells $(1\,2)$ or interchanging the outer bells $(2\,3)$. The permutation $(1\,3)$ is not allowed because bells cannot move more than one position between changes. For $n = 4$ bells, list all the allowable moves as permutations.

6. How many allowable moves are there for $n = 5$ bells? List all of them in permutation form (eg. $(1\,2)$) List all the allowable permutations for $n = 6$ bells.

7. By looking for a pattern in your answers to the previous two problems, make a table with the number of allowable moves for n bells. One column should be the number of bells n and the other column should be the number of allowable moves. Begin with $n = 2$ and 1 move, then $n = 3$ and 2 moves, etc. The last row of your table should be $n = 12$.

8. Examine the sequence of changes called **Canterbury Minimus**, listed at the bottom of page 2 of the class handout on change ringing. Which of the six rules for an extent does it satisfy? Explain. Is this a legitimate extent?

9. What are some similarities and differences between Canterbury Minimus and **Plain Bob Minimus**?

For the remaining questions, let $a = (1\,2)(3\,4), b = (2\,3), c = (3\,4), d = (1\,2)$ be four possible permutations of four bells. Recall that Plain Bob Minimus can be written in terms of the permutations which create it as $[(ab)^3ac]^3$.

10. What is the permutation $(abcd)^2$ equivalent to? Why couldn't this be used as part of an extent?

11. Express Canterbury Minimus in terms of the permutations which create it.

12. Write out the first 25 rows of the extent on $n = 4$ bells determined by $[dbadabdc]^3$. This extent is called **St. Nicholas Minimus**. Does Bell 1 go **plain hunting**?

13. Write out the first 25 rows of the extent on $n = 4$ bells determined by $[(db)^2da]^4$. This extent is called **Erin Minimus**. Does Bell 1 go plain hunting?

14. You are the composer! Make up your own extent, satisfying at least the first three rules, on $n = 4$ bells that is different from Plain Bob Minimus, Canterbury Minimus, St. Nicholas Minimus, Erin Minimus or Reverse Bob Minimus. List all 25 rows as well as the permutation form of your extent. Try to make Bell 1 go plain hunting in your composition. Does your extent satisfy Rule 6, the "palindrome property?"