MATH 110: Mathematics and Music
Homework Assignment #4
DUE DATE: Fri., Feb. 24th, start of class.

You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your own work.

Note: Please list the names of any students or faculty who you worked with on the assignment.

1. Read Chapter 3 of the course text, *The science of musical sound*, by Charles Taylor.
2. According to Taylor, what role does the brain play in our ability to hear music? Give some examples.
3. Why is it that our brain can tell the difference between an electronic synthesized version of A 440 and the same note sounded by an oboe, even if their corresponding wave-forms are virtually identical?
4. Recall that 2π radians is equivalent to $360°$. Evaluate the following without a calculator. Be sure to explain how you obtained your answers.

   a. $\sin(50\pi)$ **b.** $\sin(\frac{9\pi}{2})$ **c.** $\cos(51\pi)$ **d.** $\cos(k\pi)$ where k is an odd integer

5. Given a sound wave of the form

   $$y = 22\sin(1000\pi(t - 200))$$

   where t is measured in seconds, what is the amplitude, period and frequency of the sine wave? Could a dolphin hear this note?

6. On the same set of axes, sketch the graph of the two sine waves

   $$y = 5\sin(2t) \qquad \text{and} \qquad z = 5\sin(2(t - \frac{\pi}{2})).$$

   You might draw one curve with a solid line and the other dashed, or use different colors.

7. A piano tuner comparing two of three strings on the same note of the piano hears three beats a second. If one of the two notes is E (330 Hz) above middle C , what are the possibilities for the frequency of the other string?
8. Using the trig addition formula for sine derived in class, simplify the sum

   $$\sin(310\pi t) \ + \ \sin(318\pi t).$$

   What is the "frequency" of the resulting wave and how many beats per second would you expect to hear if the two waves were sounded together?

9. Recall that

$$\begin{aligned} \sin(A+B) &= \sin A \cos B + \cos A \sin B \quad (1) \\ \cos(A+B) &= \cos A \cos B - \sin A \sin B \quad (2) \end{aligned}$$

a. Set $A = B = \theta$ in equation (1) and derive the double-angle formula for the sine function, $\sin(2\theta) = 2\sin\theta\cos\theta$.

b. Set $A = B = \theta$ in equation (2) and derive a double-angle formula for the cosine function, $\cos(2\theta) = 2\cos^2\theta - 1$. Note: $\cos^2\theta = (\cos\theta)^2$.

c. Find a formula for $\cos(3\theta)$ in terms of only $\cos\theta$ and higher powers of $\cos\theta$. *Hint:* Somewhere in your calculation you will need to use the famous identity between $\cos^2\theta$ and $\sin^2\theta$.

10. Following the proof we did in class for the sum $\sin u + \sin v$, prove a similar formula for the sum of two cosines:

$$\cos u + \cos v = 2\cos(\frac{u+v}{2})\cos(\frac{u-v}{2})$$

11. Recall that the **overtone series** of a frequency u consists of the sequence of higher and higher multiples of u: $u, 2u, 3u, 4u, 5u, \ldots$

a. List the first ten frequencies (including the fundamental) in the overtone series for A 220 Hz.

b. List the first ten frequencies (including the fundamental) in the overtone series for A 440 Hz.

c. Circle the frequencies which A 220 and A 440 have in common. These two notes are an octave apart.

d. True or False: (provide justification) Each frequency in the overtone series for a note an octave higher than u is part of the overtone series for u.

12. Recall that a note of frequency u and a note of frequency $\frac{3}{2}u$ are a perfect fifth apart.

a. List the first ten frequencies (including the fundamental) in the overtone series for $\frac{3}{2}u$.

b. Circle all of the frequencies in your series from part **a.** which are also contained in the overtone series for u.

c. Make a bold conjecture concerning the overlap of the two overtone series. Why does this reinforce the idea that the perfect fifth is one of the most consonant intervals after the octave?