Topics in Mathematics: Mathematics and Music

MATH 110, MWF 11:00 - 11:50, Brooks 132, Spring 2006

Professor Gareth Roberts

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Office hours: Mon. 10:00 - 10:50, Tues. 10:30 - 12:00, Wed. 12:00 - 1:00, Fri. 8:00 - 8:50 or by appointment.

Required Text: *Music and Mathematics: From Pythagoras to Fractals*, edited by Fauvel, Flood and Wilson. You should also purchase the staff paper *Music Tablet*, available at the college bookstore.

Web page: `http://mathcs.holycross.edu/~groberts/Courses/MA110/homepage.html` Homework assignments, handouts, schedule changes, exam materials, useful links and other important information will be posted at this site. Please bookmark it!

Syllabus: The connections between mathematics and music, from both a structural and aesthetic viewpoint, are plentiful. Both use a specialized form of notation to communicate their ideas. Each subject has its own logical structure and set of axioms finely tuned over centuries of study. Students in high school geometry learn the axiomatic technique of Euclid to write their first proofs. Students in musical theory learning to compose for four voices are taught to avoid parallel fifths and octaves just as Bach did in his choral works. Mathematicians use numbers as the invariant building blocks of their theory as musicians use pitch as the common denominator of their creations. Just as the number three has the same abstract meaning to mathematicians everywhere, the concert A440 used to tune an orchestra is the same regardless of location.

There are aesthetic, less concrete connections to be considered as well. Both fields have produced great child prodigies (for example, Mozart and Gauss). Parents play Mozart and Bach to their children, even in the womb, to help foster their brain development and analytic skills. Many mathematicians are outstanding musicians, while many musicians, in particular composers, possess sharp mathematical minds. Scholars often speak of the "beauty" and "purity" of mathematics, although the same lofty descriptions could equally apply to music. While music has the obvious capacity to move the spirit, great mathematical discoveries and insights are often accompanied with an overwhelming sense of elation. The great mathematician Andrew Wiles wept on camera while speaking about his incredible proof of Fermat's Last Theorem.

Finally there are the composers and musicians who, whether they are cognizant of the fact or not, use mathematical concepts in their creations. Bach frequently used mathematical operations such as translations and reflections in his fugues to create wonderfully rich counterpoint. Bell ringers in towers throughout England have been using permutations, symmetry and group theory in change ringing (sometimes called campanology) to announce important events since the early 1600's. Music theorists and mathematicians alike have claimed the appearance of the golden section in compositions of some of the great composers (Mozart, Debussy and Bartók to name a few). The modern composer Xenakis used calculus and the theory of probability to create his "stochastic music."

This course will explore the dynamic relationship between mathematics and music. The pertinent mathematical and musical concepts will be developed as the need arises. Musical topics to be considered include basic music theory (pitch, scales, intervals, rhythm, chords, progressions), non-standard tunings and scales, the overtone series, and melodic and harmonic structure. The mathematical topics to be studied include graphs, logarithms, equivalence relations, modular

arithmetic, group theory, rational and irrational numbers, symmetry, probability and discrete mathematics.

A tentative outline of the course is given below. Although we will cover material from many chapters in the course text, there will also be handouts used for several topics. In addition, a few days have been reserved for course labs using a device called a monochord.

- Introduction: the many connections between mathematics and music (2 classes)
- Basic Music Theory: notation, pitch, intervals, scales, keys, rhythm (5 classes)
- The Science of Music: a vibrating string, the wave equation, the sine function, frequency versus length (monochord), the overtone series (5 classes)
- Tuning and Temperament: the Pythagorean scale, equal temperament, rational and irrational numbers, alternative tunings and scales (5 classes)
- Exam I
- Change Ringing: Bell ringing in England, permutations, group theory (5 classes)
- Sulski Lecture: March 22nd, 8:00 - 9:00 pm, "The Sound of Algebra," Prof. Michele Intermont HC '89
- Musical Group Theory: transformations of the plane, more group theory, symmetry, Bach, counterpoint and the fugue (6 classes)
- Guest Lecture: April 7th, "Symmetry in Music," Prof. John Little
- Exam II
- Modern Music: Bartók, Schoenberg, Davies, Xenakis (5 classes)
- Final Project presentations (3 classes)
- Final Exam (Cumulative)

Course Objectives:

1. Investigate the multiple connections between mathematics and music.
2. Develop skills in critical thinking and abstract reasoning.
3. Develop an understanding of musical theory and a deeper appreciation for music.
4. Integrate your artistic and analytic skills.

Homework: Homework will be assigned on a regular basis. Assignments will be posted on the course web page. While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. Take care to write up solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of the Departmental Policy on Academic Integrity.

NOTE: LATE homework will NOT be accepted. The only excused homework which is late will be accompanied by a letter from your Class Dean. However, you will be allowed ONE "mulligan" over the course of the semester where you can turn in the assignment up to one week after the original due date.

Concert Reviews: You are required to attend two musical performances during the semester and turn in a typed, 1-2 page review of the concert. The purpose of these reviews is to enhance your musical appreciation, to support your fellow students and the arts, and to witness the possible connections to our subject material. The reviews will count toward your homework grade. A list of performances will be distributed shortly.

Final Project: You will be expected to complete a final project consisting of a musical composition and performance demonstrating some of the mathematical concepts learned in the course. A brief, typed report should accompany your composition explaining the mathematical connections and rationale in your work. The performances will take place during the last three class periods.

Exams: There will be 2 midterm exams (in class) and a comprehensive final at the end of the semester. The exam schedule is given below. Please make a note of these dates and plan accordingly. Any conflicts must be legitimate and brought to my attention well before the exam is scheduled. If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to contact Dr. Matthew Toth of Disability Services in Hogan 207 (x 3693) to obtain documentation of your disability.

	Exam 1	**Wed., March 1**	**11:00 - 11:50 am**
Exam Schedule:	**Exam 2**	**Wed., April 12**	**11:00 -11:50 am**
	Final	**Sat., May 6**	**8:30 - 11:30 am**

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. Anyone who violates the Departmental Policy on Academic Integrity will receive a 0 for that assignment as well as possible further disciplinary action involving your Class Dean.

Grade: Your course grade will be based on the following breakdown:

- homework assignments and concert reviews 35%
- midterm exams 30%
- final exam 25%
- final project 10%

How to do well in this course:

- Attend class, participate and ask questions.
- Do your homework regularly.
- Work with your classmates.

> *May not Music be described as the Mathematic of Sense, Mathematics as the Music of reason? The soul of each the same! Thus the musician feels Mathematic, the mathematician thinks Music, — Music the dream, Mathematic the working life, — each to receive its consummation from the other.*
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> James Joseph Sylvester, 1865