MATH 136-04 AP Calculus

Sample Final Exam **Fall 2010** **Prof. G. Roberts**

There are a total of 200 points on the exam.

1. [20 pts.] Find dy/dx for each of the following functions. Simplify your answer as best as possible.

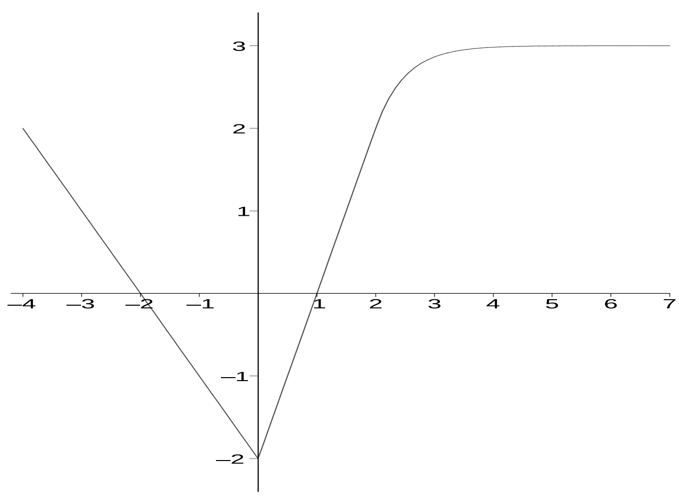
   (a) $y = x^2 \tan(\sin x)$

   (b) $y = \dfrac{1}{\sqrt{x^4 + 10}}$

   (c) $y = x^3 + 3^x$

   (d) $y = x^{x^2}$

2. [25 pts.] The graph of a function $f(t)$ is shown below. This function has a horizontal asymptote at $y = 3$. Define the function $F(x) = \displaystyle\int_{-4}^{x} f(t)\,dt$ for $x \geq -4$.



   (a) Sketch the graph of the derivative $f'(t)$ over the domain $-4 \leq t \leq 7$.

   (b) Find $F(-2)$ and $F(0)$.

   (c) Find $F''(0)$ if it exists. If it does not exist, explain why.

   (d) Sketch a graph of $F(x)$ over $-4 \leq x \leq 7$.

   (e) What type of function (constant, linear, quadratic, trig, exponential, etc.) does $F(x)$ resemble as $x \to \infty$?

3. [25 pts.] Suppose that $f(x) = \dfrac{3x}{x^2+4}$.

   (a) Find the vertical and horizontal asymptotes of $f(x)$.

   (b) Calculate and simplify $f'(x)$ and $f''(x)$.

   (c) Locate and classify any critical points of f.

   (d) Locate any inflection points.

   (e) Sketch the graph of $f(x)$.

4. [20 pts.] Evaluate the following integrals. You must use a valid integration technique to receive credit.

   (a) $\displaystyle\int x^2 \cos x \, dx$

   (b) $\displaystyle\int \frac{t+2}{t^2+4t+8}\, dt$

   (c) $\displaystyle\int \frac{x+13}{x^2-2x-3}\, dx$

   (d) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+1}}\, dx$ *Hint:* Use the trig. sub. $x = \tan\theta$.

5. [15 pts.] Determine whether the given infinite series converges or diverges using any of the tests discussed in class. You must provide a valid reason to receive full credit.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^4}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n(n+1)}{n-1}$

6. [20 pts.] Consider the initial-value problem

$$\frac{dy}{dx} = (2-y)x^2, \qquad y(0) = 1$$

   (a) Use Euler's method with a step-size of $\Delta x = 0.25$ to estimate the value of $y(1)$. (Round to the fourth decimal place.)

   (b) Solve the differential equation subject to the given initial condition.

   (c) What is the actual value of $y(1)$? Compute the error of the Euler's method estimate in part **(a)**.

7. [13 pts.] Recall Newton's Law of Cooling: The rate at which the temperature of an object cools is proportional to the difference in temperature between the object and its surrounding medium. A hot cup of coffee initially at 99°C is left in a room where the temperature is 20°C. If the coffee cools to 90°C in 2 minutes, find the temperature of the coffee after 5 minutes. How long will it take for the coffee to reach a drinkable temperature of 60°C? Assume that the temperature of the room is held constant at 20°C.

8. [30 pts.] TRUE or FALSE: If true, provide a brief explanation. If false, give a counterexample to the statement.

   (a) If $f(x)$ is a strictly decreasing function, then its inverse $f^{-1}(x)$ is also a strictly decreasing function.

   (b) The following limit does not exist:

   $$\lim_{t\to 0} \frac{\cos(t^2) - 1}{t^4}$$

   (c) If $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n \cdot b_n$ diverges.

9. [32 pts.] Some conceptual questions:

   (a) Use the limit definition of the derivative to find $f'(x)$ for $f(x) = \sqrt{x+1}$.

   (b) Derive the formula for the volume of a sphere of radius r by rotating the top half of the circle $x^2 + y^2 = r^2$ about the x-axis.

   (c) Find the sum of the series $\sum_{n=2}^{\infty} \frac{4}{n^2 - 1}$. *Hint:* Master and Commander

   (d) For what values of k is $y(x) = e^{kx}$ a solution to the second-order differential equation $y'' - 3y' - 10y = 0$.