MATH 136-04, Fall 2010

Rates of Change in the Natural and Social Sciences (Section 3.8)

Physics: If $s(t)$ is the position of a moving object (or particle on a line) as a function of time t, then $s'(t) = v(t)$ is the instantaneous **velocity** and $s''(t) = v'(t) = a(t)$ is the **acceleration**.

Example 1: A ball thrown vertically upward has a height (position) after t seconds given by $s(t) = 80t - 16t^2$.

a) What is the maximum height reached by the ball?

b) How long is the ball in the air?

c) What is the initial velocity of the ball?

d) What is the velocity of the ball when it strikes the ground?

e) What is the velocity of the ball when it is 96 ft. above the ground on its way up? on its way down?

Biology: If $P(t)$ is the population of a given species (people, animals, bacteria, etc.) as a function of time t, then $P'(t)$ is the instantaneous **growth rate** of the population. Thus, if $P'(t) > 0$, the population is increasing at time t and if $P'(t) < 0$, the population is decreasing at time t. Strictly speaking, P is usually a discontinuous step function (set of data points), so we interpolate the values in between to create a smooth approximating curve that is differentiable.

Example 2: The population of a species of rabbits in a town is modeled by

$$P(t) = \frac{5e^{4t}}{4 + e^{4t}},$$

where t is in years and P is in thousands.

a) Show that the population is always increasing in size.

b) What is the long-term fate of the population? In other words, what is $\lim\limits_{t \to \infty} P(t)$?

c) What is $\lim\limits_{t \to -\infty} P(t)$?

d) Using parts **a)**, **b)** and **c)**, sketch the graph of $P(t)$.

e) At what time is the rabbit population growing the fastest? In other words, when does P' have a maximum? How fast is the population growing at this time?