MATH 136-01, Fall 2004

Computer Lab #5

Slope Fields

DUE DATE: Tuesday, Dec. 7th, 5:00 pm.

The goal for this lab project is to further your understanding of the geometric and qualitative concepts utilized in the study of differential equations. Since most differential equations cannot be explicitly solved to obtain an analytic solution (formula), this is an important aspect of the subject. You will use MAPLE to obtain slope fields for differential equations and graph solution curves. Ultimately, you will gain a better understanding of the relationship between the function defining a differential equation and the geometry of the solutions to that equation. This is often described as *qualitative analysis* to researchers using differential equations.

It is **required** that you work in a group of two or three people. Any help you receive from a source other than your lab partner(s) should be acknowledged in your report. Please turn in one report per group, listing the names of the groups members at the top of your report. Be sure to answer all questions carefully and neatly, writing in complete sentences. You should turn in your answers and graphs on separate sheets of paper.

Slope Fields

In class we have discussed one geometric technique of understanding a differential equation, namely the *slope field.* Given a differential equation

$$\frac{dy}{dx} = f(x,y)$$

we can plot a tiny slope segment at each point (x, y) in the plane whose slope is given by the function value at that point, $m = f(x, y)$. Using a computer, we can obtain a picture of the solution curves for the differential equation by plotting hundreds of slope marks on the xy-plane, obtaining the slope field which solutions much follow. Each solution curve must be tangent to the slope field at every point. Think of driving a car in a large parking lot covered with arrows. At every instant there is a new arrow telling you where to go. The path of your car traces out a particular solution curve.

MAPLE can draw slope fields using the `DEplot` command from the `DEtools` package. For example, to plot the slope field for the equation

$$\frac{dy}{dx} = x + y$$

you type the following:

```
with(DEtools):
deq := diff(y(x),x) = x + y(x);
DEplot(deq, y(x), x=-2..2, y=-3..3);
```

The first command loads the `DEtools` package (you only need to run this once per MAPLE session). The second command above defines the differential equation. **NOTE:** Whenever the dependent variable is entered on the right-hand side, it must be entered as `y(x)` rather than just plain `y`. If we wanted $\sin y$ for example, we would have to type `sin(y(x))`. The `DEplot` command is used to actually draw the slope field where the x-range is given by `x=-2..2` and the y-range is given by `y=-3..3` . (You may want to change these depending on the differential equation.)

There are some other commands which may be useful to you. If you want to draw a solution curve superimposed onto the slope field, you use the following:

```
ics := [ [0,1], [0,-1] ];
DEplot(deq,y(x),x=-3..3,y=-3..3,ics,dirgrid=[30,30],stepsize=.01,linecolor=[blue,black]);
```

This should give the slope field for $dy/dx = x + y$ and two solution curves (the first in blue, the second in black) through the points $y(0) = 1$ and $y(0) = -1$. Why does the black solution curve appear to be a straight line?

The first command above lists the initial conditions for your differential equation, in the form $[x_0, y_0]$. These must be included as a list, with brackets around the entire list. In the second command are some additional options you might like to use. Placing `ics` into the `DEplot` command tells MAPLE to draw solution curves on top of the slope field. The `dirgrid=[30,30]` option is an array of two integer values, specifying the number of horizontal and vertical mesh points to use for arrows. (So in this case we would have roughly 900 little slope segments.) For this command, [2,2] is the minimum, and [20,20] is the default value. The `stepsize=0.01` option tells MAPLE to take steps of length `stepsize=0.01` between stored points. The lower the stepsize, the better approximation to the solution MAPLE obtains, but the longer it takes to create it. (Be careful not to crash your computer!) Finally, the `linecolor=[blue,black]` option tells MAPLE to draw your two solution curves in blue and black respectively. This makes the curves dark enough to be visible upon printing. If there were three initial conditions and thus, three curves, you would need to specify three colors, etc. Play around with the values of these options so you can learn how they effect the figure generated by MAPLE.

Be forewarned that the solutions MAPLE draws may not always be correct or even useful. It is not hard to introduce numerical error when approximating the solution to a differential equation. If you would like to learn more about the `DEplot` command and its options, you can type `?DEplot` and follow MAPLE's built-in help window.

Exercises:

For each of the following differential equations (1-4):

a. Use MAPLE to draw the slope field (you choose an appropriate range).

b. Use MAPLE to graph the solution curves corresponding to the initial conditions $y(0) = 1, y(1) = 0, y(0) = -1$ on your slope field. Turn in a printout with the the slope field and solution curves on the same plot.

c. Describe the long-term behavior of each of the three solution curves from part **b.** For example, does a solution curve approach a particular value as $x \to \infty$? Does it oscillate? Does it tend to $+\infty$ or $-\infty$? Be as specific as possible.

d. Using any of the ideas or techniques discussed in class, find a formula for the general solution (simplify as best as possible). For one of the examples, this is impossible.

e. For the examples where you can find a formula for the general solution, give the particular solution to the differential equation which satisfies $y(0) = 1$. (In other words, find the value of the integration constant c.) Your answer should agree with your plot from part **b.**

1.
$$\frac{dy}{dx} = x^2 y^2$$

2.
$$\frac{dy}{dx} = y(y-1)$$

3.
$$\frac{dy}{dx} = \cos(x^2-2)$$

4.
$$\frac{dy}{dx} = x \sin y$$

5. Guess the slope field. Given the following slope fields, find the function $f(x, y)$ for the right-hand side of the differential equation
$$\frac{dy}{dx} = f(x, y)$$
which yields the given slope field. All you need to do for each slope field is to state the function $f(x, y)$. *Hint:* First look for the obvious indicators, such as equilibrium points. Second, consider the geometric shape in the figure for which the slope marks appear to be constant. Check your answers using MAPLE.

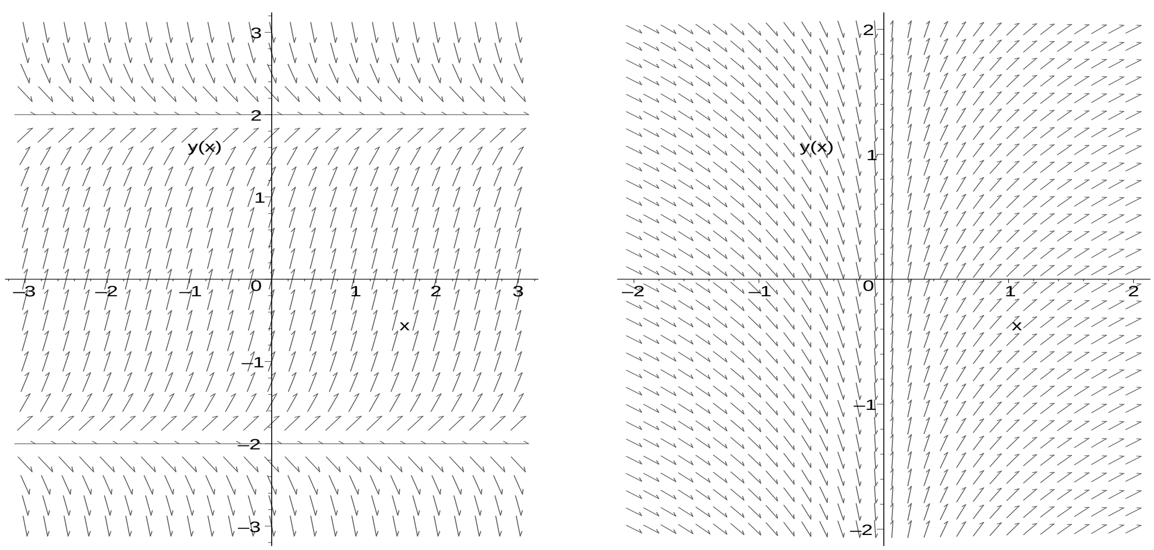

Figure 1: Slope fields **a.** and **b.** Guess the right-hand side $f(x, y)$ of the differential equation $dy/dx = f(x, y)$ which yields the given slope field.

TURN OVER

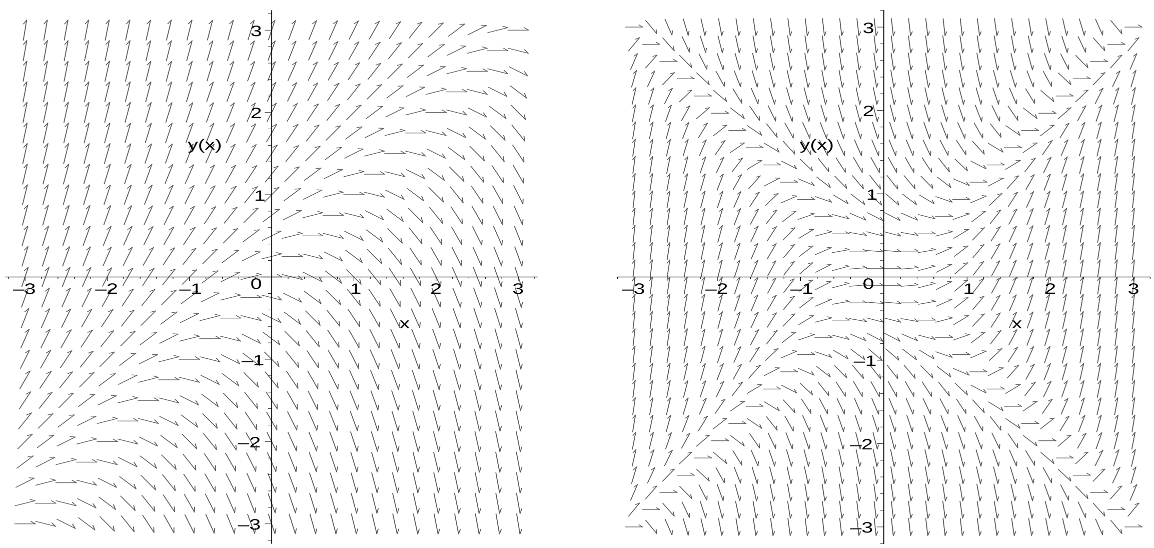

Figure 2: Slope fields **c.** and **d.** Guess the right-hand side $f(x, y)$ of the differential equation $dy/dx = f(x, y)$ which yields the given slope field.