MATH 136-01, Fall 2004

Computer Lab #1

Using MAPLE to Visualize Functions

DUE DATE: Monday, Sept. 20th, in class.

The goal of this lab project is for you to become familiar with the software package MAPLE. You should have already read the *Introduction to MAPLE Computer Labs* handout. In this lab, you will use MAPLE to plot and evaluate functions and to compare new functions generated from old ones (via shifting, stretching, etc.) The lab is designed as a review of material from Chapter 1 of the course text.

It is **required** that you work in a group of two or three people. Any help you receive from a source other than your lab partner(s) should be acknowledged in your report. For example, a textbook, web site, another student, etc. should all be appropriately referenced. Please turn in one report per group, listing the names of the groups members at the top of your report. Be sure to answer all questions carefully and neatly, **writing in complete sentences.**

You should turn in your answers on separate sheets of paper. There are **four plots** to be turned in for this report. You should not turn in any other graphs unless they significantly contribute to the explanation of an answer. For example, it is ok to say, "By plotting the graph of $f(x) + 2$, we see that adding 2 to the function shifts the graph upward 2 units," without actually turning in the graph.

1. Use MAPLE to define the polynomial $p(x) = x^3 - 8x - 3$ and then plot it over the domain $-5 \leq x \leq 5$. The commands for this are listed below. Be sure to type them in exactly as below.

```
p := x -> x^3 - 8*x - 3;
plot(p, -5..5);
```

   (a) Print out and turn in a graph of $p(x)$ over the domain $-5 \leq x \leq 5$.

   (b) Find all the **roots** (zeroes) of $p(x)$. By zooming in on your graph, estimate them as best as you can using MAPLE. What are the *exact* values of the roots?

   (c) Use MAPLE to find the number of digits of $p(1,000,000)$.

   (d) By adjusting the plot range of the graph, find the limits

$$\lim_{x\to\infty} p(x) \qquad \text{and} \qquad \lim_{x\to-\infty} p(x).$$

   You can do these without a computer of course, but the point here is to learn to use MAPLE.

   (e) Define a new function $g(x) = p(x + 3)$ using MAPLE with the command

```
g := x -> p(x+3);
```

   How does the graph of g differ from the graph of p? What domain should you plot the graph of g over to obtain the same **exact** graph as you printed out in part (a)?

(f) Plot both functions p and g on the same set of axes choosing an appropriate range for the x and y-axis. See the *Introduction to MAPLE Computer Labs* handout on how to plot multiple functions. Print out your plot and make sure to label each graph.

(g) You can also do algebra with MAPLE. Type the command `expand(g(x));` to find the expansion in powers for $g(x)$. How could you have predicted ahead of time that $g(x)$ has no constant term?

2. Use MAPLE to define the functions $f(x) = x^2 - 3x + \sin x$, $g(x) = f(-x)$ and $h(x) = -f(x)$. (See the syntax from the previous problem.)

   (a) How many roots does the function $\sin x$ have? How many roots does $f(x)$ have? Explain. Why are they different?

   (b) How does the graph of g differ from the graph of f? In general, what effect does replacing x by $-x$ have on the graph of $f(x)$?

   (c) How does the graph of h differ from the graph of f? In general, what effect does replacing $f(x)$ by $-f(x)$ have on the graph of $f(x)$?

   (d) Plot all three functions f, g, h on the same set of axes choosing an appropriate range for the x and y-axis. (Be an artist!) Print out your plot and make sure to label each graph.

3. In class we stated that exponential functions always grow faster than power functions. It may take a while to happen, but the graph of an exponential curve will eventually surpass that of x^n, no matter how large the value of n. Consider the functions $f(x) = 2^x$ versus $p(x) = x^{10}$. Initially, p is much larger than f. For example, $p(2) = 1,024$ while $f(2) = 4$.

   (a) Where do the graphs of f and p cross? (Give the x-value and assume $x > 2$.) Explain how you found your answer.

   (b) Print out a plot of both f and p on the same set of axes with the domain chosen so as to nicely illustrate the crossing found in part (a). Be sure to label each graph.

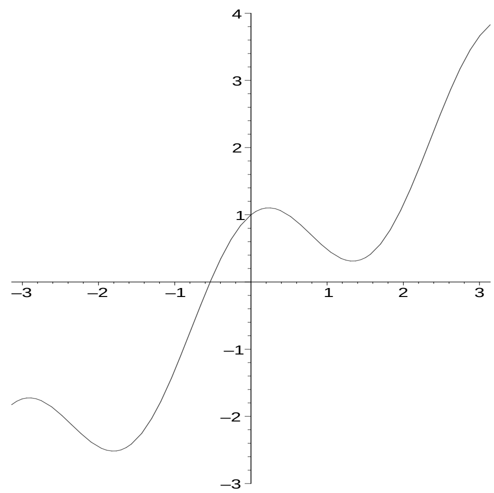

Figure 1: The graph of $f(x) = \cos(2x) + 0.9x$.

4. **What's the function?**

   The graph of $f(x) = \cos(2x) + 0.9x$ is shown above in Figure 1 plotted over the domain $-\pi \leq x \leq \pi$. Using your knowledge on shifting, stretching and reflecting functions, find formulas for the functions $g(x)$ and $h(x)$ shown in Figure 2. Express your answers in terms of $f(x)$ (for example, $g(x) = 100f(\pi x - \ln(3))$ is a possible answer.)

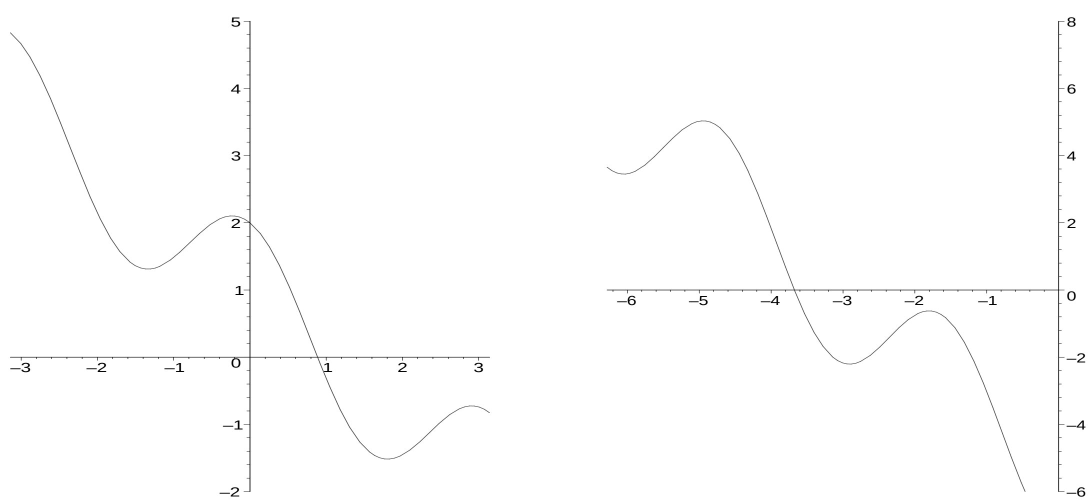

Figure 2: The graphs of $g(x)$ (left) and $h(x)$ (right).