MATH 136-01
Chapter 6 Topic Review Sheet

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This is a list of terminology and topics covered in the sixth chapter of *Calculus*, D. Hughes-Hallet, et al. 3rd edition. Please consult the text for full definitions, statements of properties, and numerous examples and exercises. Terms in bold face are defined in the text. The primary focus of Chapter 6 is on how to construct antiderivatives using graphs, numerical information or explicit formulas.

Antiderivatives Graphically and Numerically (Section 6.1) We call $F(x)$ an **antiderivative** of $f(x)$ if $F'(x) = f(x)$. This is an inverse process, the opposite of finding the derivative of a function. Here we seek the function F whose derivative is f. Given the graph of F', we can use the properties of the derivative developed earlier in the course to sketch the graph of an antiderivative F. For example, if F' is positive, then F is increasing.

Note that there is not a unique antiderivative for a given function but rather a **family of antiderivatives**. However, if we are given a specific value for the antiderivative, such as $F(0) = 10$, then this determines a unique antiderivative. The Fundamental Theorem of Calculus is particularly valuable for finding specific values of an antiderivative F. If we know the area under F' and the value $F(a)$, then we can use

$$F(b) - F(a) \;=\; \int_a^b F'(x)\,dx$$

to find $F(b)$.

Constructing Derivatives Analytically (Section 6.2) Certain antiderivatives are straight-forward to find analytically using our knowledge about derivatives. For example, the antiderivative of $F'(x) = 0$ is simply $F(x) = c$ where $c \in \mathbb{R}$ is an arbitrary **integration constant**. This is true because the derivative of a constant is zero. The antiderivative of $F'(x) = k$ is the linear function $F(x) = kx + c$ since the slope of F is k. Another "easy one" is to use the power rule in reverse:

$$\int x^n\,dx \;=\; \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$

If $n = -1$, then we use $(\ln x)' = 1/x$ to derive

$$\int \frac{1}{x}\,dx \;=\; \ln|x| + c.$$

The absolute value is necessary here since the domain of our antiderivative should match that of our derivative which in this case is $\{x \in \mathbb{R} : x \neq 0\}$. The derivatives of e^x, $\cos x$ and $\sin x$ all lead to the following antiderivatives:

$$\begin{aligned} \int e^x\,dx &= e^x + c \\ \int \cos x\,dx &= \sin x + c \\ \int \sin x\,dx &= -\cos x + c \end{aligned}$$

Finally, from the fact that differentiation is a linear operation, we have that the antiderivative of a sum (or difference) of two functions is the sum (or difference, respectively) of their antiderivatives. Likewise, the antiderivative of a constant times a function is the constant times the antiderivative of the function.

Second Fundamental Theorem of Calculus (Section 6.4) Any continuous function has an antiderivative. We may not be able to find an explicit formula for it, but we know it exists by the **Second Fundamental Theorem of Calculus**. This theorem states that if f is continuous on an interval and a is any number in that interval, then the function F defined by

$$F(x) \;=\; \int_a^x f(t)\,dt$$

is an antiderivative of f. In other words, $F'(x) = f(x)$.

It is sometimes useful to visualize the function $F(x)$ as an "area function," finding the area under f from the numbers a to x. As x varies, the area varies and therefore $F(x)$ changes. The Second Fundamental Theorem of Calculus states that the derivative of this area function $F(x)$ exists and is given by the "height," the value of the function f.