MATH 136-01

Chapter 1 Topic Review Sheet

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This is a list of terminology and topics covered in the first chapter of *Calculus*, D. Hughes-Hallet, *et. al.* 3rd edition. Please consult the text for definitions, statements of properties, and numerous examples and exercises. Terms in bold face are defined in the text. You are responsible for this material **regardless of whether it was actually discussed in class.**

Basic Function Terminology The focus of this course is real-valued functions of a real variable. That is, functions whose **domain** and **range** are subsets of the real numbers (or real line). Note that the range depends on the expression or rule for the function *and* the domain of the function. We use $\mathbb{R}$ to denote the real number line and $f : A \to \mathbb{R}$ to denote a function whose domain is the subset A of $\mathbb{R}$ and whose range is contained in $\mathbb{R}$.

Representation of Functions We will use tables, graphs, symbolic expressions or formulas, and prose descriptions to define and describe functions.

The primary content of *Chapter 1: A Library of Functions* is a catalogue of functions that will be used in calculus and that you should have seen in your high school calculus course. The functions are introduced in an elementary manner that does not employ differentiation or integration. Here is a list of functions and particular terminology that is used to describe the functions.

Linear Functions (Section 1.1) A function $y = f(x) = mx + b$, where m and b are constants is called a **linear function**. The constant m is called the **slope** and the constant b is called the **y-intercept** or **vertical intercept** of f. A linear function is **increasing** if its slope is positive and decreasing if its slope is negative. When $y = mx$, that is, when $b = 0$, we say that y is **directly proportional** to x.

Exponential Functions (Section 1.2) A function $y = P(t) = P_0 a^t$, where $a > 0$ and P_0 are constants, is called an **exponential function**. The constant P_0 is called the **initial quantity** and a is called the **base**. When $P_0 > 0$, we have **exponential growth** when $a > 1$ and **exponential decay** when $a < 1$. We will pay particular attention to the **half-life** when there is exponential decay and the **doubling time** when there is exponential growth. In either case, the graph of P is **concave up**. When $P_0 < 0$ the graph of P is **concave down.**

The natural base, $e = 2.71828\ldots$, is of particular importance and will be used frequently throughout the course. The equality $a^t = e^{\ln(a)t}$ allows us to convert any exponential function to one expressed in terms of e in the form $P_0 e^{kt}$. The coefficient k is called the **continuous rate of growth** or **decay**.

New Functions from Old (Section 1.3) **Composition** of functions allows us to produce a new function from two other functions. If $y = f(x)$ and $z = g(y)$, we define the **composite function** h by $z = h(x) = g(f(x))$ (where the range of f must be contained in the domain of g). Important examples are when $f(x) = x - h$ (**horizontal shift**), $f(x) = cx$ (**horizontal stretch**), $g(y) = y + k$ (**vertical shift**), and $g(y) = cy$ (**vertical stretch**). We will find it useful to know when functions are **even**, $f(-x) = f(x)$, or **odd**, $f(-x) = -f(x)$. The **inverse function** of f is the function $x = g(y)$ with the property that $g(f(x)) = x$ and is denoted $f^{-1}(x)$.

Logarithmic Functions (Section 1.4) A **logarithm** function is an inverse function of an exponential function $y = a^x$. We write $\log_a(x)$ for the inverse of a^x. It is called the **logarithm to the base** a. The **natural logarithm** or **logarithm to the base** e is denoted $\ln(x)$ rather than $\log_e(x)$. It is especially important that you be familiar with the properties of logarithms in the table on page 24. These can all be derived from properties of exponentiation.

Trigonometric Functions (Section 1.5) The definitions and properties of **trigonometric functions** are derived from relations between the lengths of arcs on a unit circle and the coordinates of points that lie on the circle. The two basic trigonometric functions are the **sine** and the **cosine**, denoted by $\sin(t)$ and $\cos(t)$ where t is measured in **radians**. The quotient is the **tangent** function, denoted by $\tan(t) = \frac{\sin(t)}{\cos(t)}$. The multiplicative inverses of these three functions are the **cosecant**, $\csc(t) = \frac{1}{\sin(t)}$, the **secant**, $\sec(t) = \frac{1}{\cos(t)}$, and the **cotangent**, $\cot(t) = \frac{\cos(t)}{\sin(t)}$.

The trigonometric functions are examples of **periodic functions**. More general periodic functions can be obtained by composing trigonometric functions with other functions and by combining them algebraically (using sum, difference, product, and quotient). The **period**, **amplitude**, and **phase difference** of a periodic function are used to describe its behavior.

We will also have occasion to use and study the inverse functions of the six trigonometric functions.

Powers, Polynomials, and Rational Functions (Section 1.6) A **power function** is a function of the form $y = f(x) = kx^p$, where $k \neq 0$ and p are constants. We will be concerned with **rate of growth** of power functions in comparison to other functions, in particular, exponentials and logarithms. A function that is a sum of power functions with natural numbers as powers is called a **polynomial function**. The general form of a polynomial is

$$y = p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

where n is a non-negative integer (zero is allowed) and the coefficients are constants. The highest power that appears, in this case n, is called the **degree** of the polynomial. The **roots** or **zeros** of p are the x-intercepts of the graph, the values x for which $p(x) = 0$. If the real number r is a root of p, then p can be written as a product, or **factored**, in the form $p(x) = (x - r)q(x)$ where q is a polynomial of degree $n - 1$. It is important to understand how the different possibilities for the roots of a polynomial are reflected in the graph of p.

A **rational function** is a quotient of polynomials and takes the form $f(x) = \frac{p(x)}{q(x)}$. The roots of $p(x)$ determine the zeros of $f(x)$ and roots of $q(x)$ determine the **vertical asymptotes** of f. It will be important to determine the asymptotic behavior of rational functions as $x \to \pm\infty$, that is, whether f has a **horizontal asymptote** as $x \to \infty$ and/or $x \to -\infty$.

Introduction to Continuity (Section 1.7) At this point in the course, we will want to have an intuitive understanding of what it means for a function to be **continuous**, that is, that the graph of the function on its domain has no breaks (it can be drawn without lifting the pen off the paper.) Later in the course, we will say more about the possibilities for a **discontinuity** of a function at a point. One useful consequence of continuity is the **Intermediate Value Theorem**.