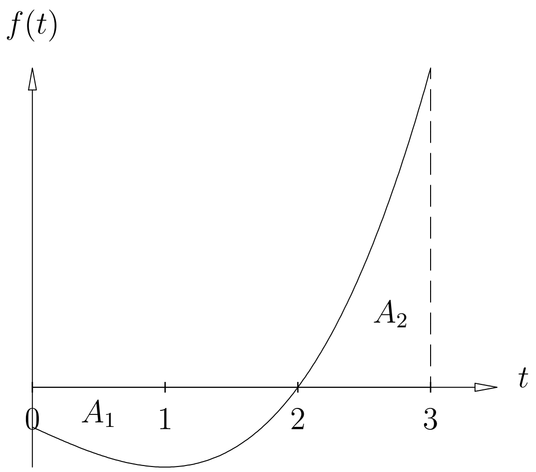
Math 132: Calculus for the Physical & Life Sciences 2

Spring 2006

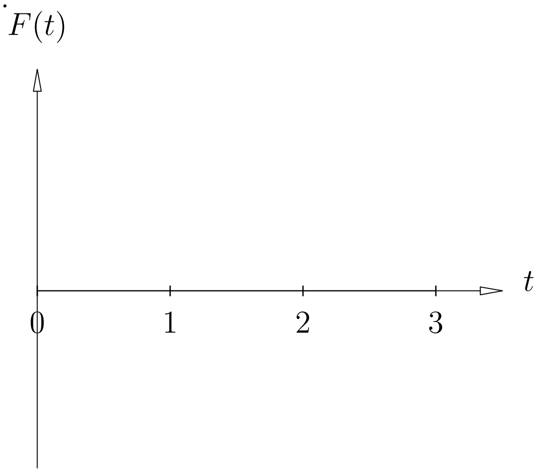
Practice Questions for the Final Exam

1. The graph of the function f is shown below. The areas shown are $A_1 = 6$ and $A_2 = 6.75$. Let F be the antiderivative of f that satisfies $F(2) = 3$.



   (a) Compute $F(0)$ and $F(3)$.

   (b) On the axes below, sketch the graph of F. Be sure to label and maxima, minima, and points of inflection.



2. Compute the following integrals. Be sure to state clearly what method of integration you are using. If you use an entry from the table of integrals, state which one.

   (a) $\displaystyle\int 2x\cos(x^2)\,dx$

   (b) $\displaystyle\int x^2\cos(2x)\,dx$

   (c) $\displaystyle\int \frac{36}{x(x-3)(x+3)}\,dx$

   (d) $\displaystyle\int \frac{1}{(4-x^2)^{3/2}}\,dx$

3. Determine whether the following improper integrals converge or diverge. Compute the value of the integral if it converges.

   (a) $\displaystyle\int_3^6 \frac{1}{\sqrt{x-3}}\,dx$

   (b) $\displaystyle\int_1^\infty \frac{1}{10000}\,dx$

4. (a) Use the trapezoid rule with $n = 2$ subintervals to estimate $\displaystyle\int_1^5 \ln x\,dx$.

   (b) Is the estimate in part (a) an overestimate or an underestimate? Explain.

5. Let R be the region bounded by $y = e^{-x}$, $y = 0$, $x = 0$ and $x = 1$.

   (a) Find the area of R.

   (b) Find the volume of the solid obtained by revolving R about the axis $y = -2$.

6. Suppose $P(t) = \frac{1}{9}t^2$ is the cumulative distribution function for the length (in inches) of a certain population of worm, for $0 \le t \le 3$.

   (a) Find the median length.

   (b) Find the proportion of the population with length at least 2 inches.

   (c) Find the density $p(x)$.

   (d) Find the mean length.

7. Find the arc length of the parametric curve $x = t^2$, $y = \frac{1}{3}t^3 + t$, $0 \le t \le 4$.

8. Both of the following series converge. One is geometric, and the other comes from evaluating a well known Taylor series. Identify which is which, and find their sums.

   (a) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k}}{(2k)!} = 1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \cdots$

   (b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k \pi^k}{4^k} = 1 - \frac{\pi}{4} + \frac{\pi^2}{16} - \frac{\pi^3}{64} + \cdots$

9. Use the indicated method to determine whether or not the series converges. You do not need to compute the sums.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1+\sqrt{n}}{n-1}$; comparison test

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+5}$; integral test

10. Use the ratio test to find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2 3^n}$$

Be sure to determine whether or not the series converges at each endpoint.

11. Let $f(x) = x^4 + x^3 + x^2 + x + 1$.

    (a) Find the third degree Taylor polynomial of f at $a = 1$.

    (b) Use the error bound for Taylor polynomials to give an estimate of the maximum possible error at $x = 1.1$.

    (c) Compute both $f(1.1)$ and $P_3(1.1)$. What is the actual error?

12. Use the Taylor series for e^x at $a = 0$ to find the Taylor series for $\dfrac{e^{x^2} - 1}{x}$ at $a = 0$. Write the series using summation notation, and write out the first four nonzero terms.

13. Use two steps of Euler's method, with $\Delta t = 0.5$ to approximate the solution of the differential equation

$$\frac{dy}{dt} = 1 + ty$$

with initial condition $y(0) = 1$.

14. (a) Solve the separable equation $\dfrac{dy}{dt} = y^2t^2$.

    (b) Find the solution which satisfies the initial condition $y(0) = 3$.

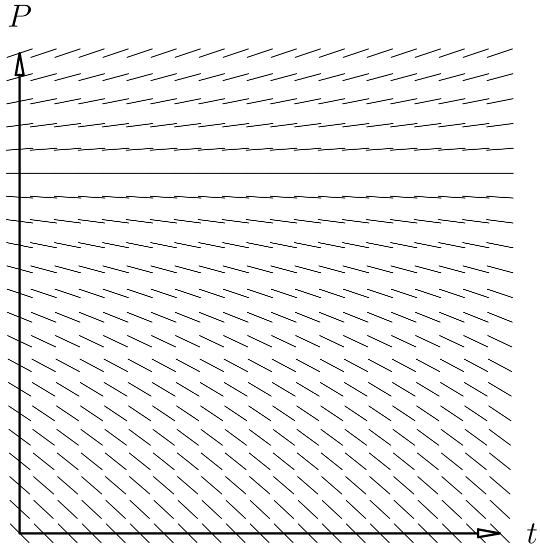
15. Suppose a population of dandelions in a yard is being harvested in such a way that it obeys the differential equation

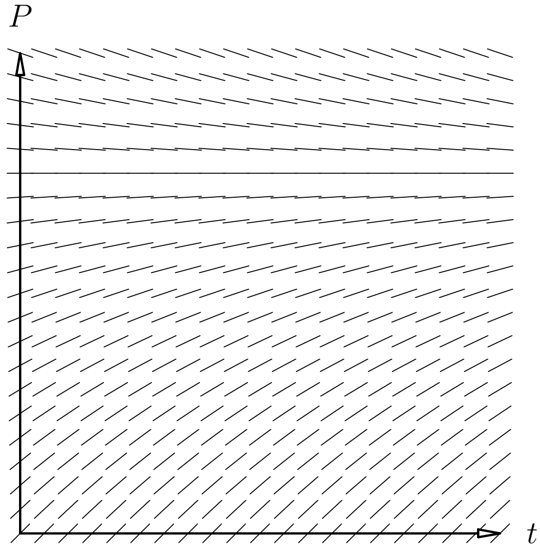
$$\frac{dP}{dt} = 0.2P - 300$$

where t is measured in weeks.

    (a) What is the equilibrium?

    (b) Identify the figure below which represents the slope field for the differential equation. Label the equilibrium.





    (c) Is the equilibrium stable or unstable?

    (d) Suppose there are initially 1200 dandelions. Will the population of dandelions:

i. be harvested into extinction?

ii. grow without bound?

iii. level off at the equilibrium?

(Circle one answer.)

16. When water drains from a tub, the rate at which the water level decreases is proportional to the square root of the water level. (This is called Torricelli's Law.)

(a) Write down a differential equation for the water level $y(t)$. Be sure to specify the sign of any constants in the equation.

(b) Solve this separable equation for y.

(c) Suppose the water level in a tub is initially 9 inches, and 5 minutes later the water level is 4 inches. How long will it take for the tub to completely drain?

17. A savings account earns 2% interest compounded continuously. Suppose withdrawals are made at a constant rate of \$1000 per year.

(a) Write down a differential equation for the account balance $B(t)$.

(b) Solve this equation for $B(t)$.

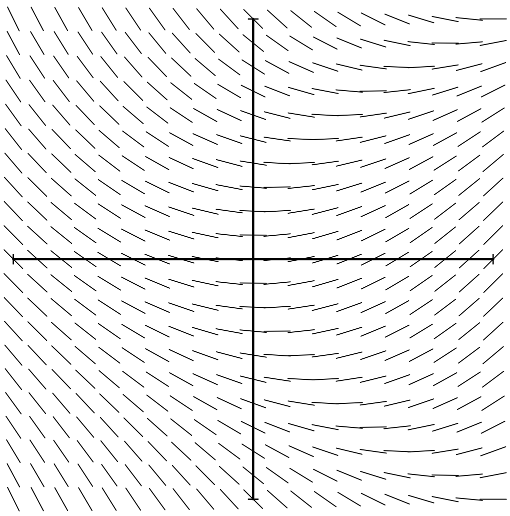
(c) If the account initially contains \$10000, when will all the money in the account be depleted? What if the account initially contains \$20000?

(d) For which initial amounts will the account never be depleted?

18. Shown below is the slope field for the differential equation

$$\frac{dy}{dt} = t - y^2$$

In the figure, both t and y vary from -1 to 1.



(a) On the axes provided, roughly sketch the solution of the differential equation with $y(0) = 1$.

(b) Compute two steps of Euler's method with $y(0) = 1$ and $\Delta t = 0.5$. Sketch the resulting approximate solution on the axes above. At $t = 1$, does Euler's method give an underestimate or an overestimate?

19. Solve each of the following separable equations.

(a) $\dfrac{dy}{dx} = e^{x-y}$

(b) $\dfrac{dy}{dt} = \dfrac{\cos t}{y+1}$, $y(0) = 2$

(c) $\dfrac{dy}{dx} = \dfrac{3y}{x}$, $y(1) = 4$.

20. For the differential equation

$$\frac{dP}{dt} = P(P-2)(P-4),$$

(a) find the equilibria.

(b) roughly sketch the slope field.

(c) classify each equilibrium as stable or unstable.