Math 132: Calculus for the Physical & Life Sciences 2

Spring 2006

Practice Questions for Midterm 3

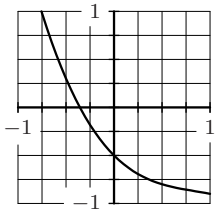
1. Suppose $f(0) = -0.5$, $f'(0) = -1$ and $f''(0) = 2$.

   (a) Write down the Taylor polynomial of degree 2 for f near $a = 0$.

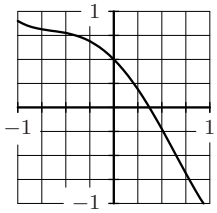
   (b) Use your answer to part (a) to estimate $f(0.3)$.

   (c) Which (if any) of the following could be the graph of f? (Recall that $f(0) = 0.5$, $f'(0) = -1$ and $f''(0) = -2$. More than one answer may be correct.)

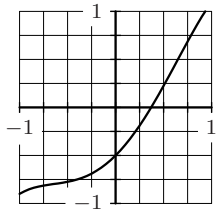
   ◯ (i) ◯ (ii) ◯ (iii) ◯ (iv) ◯ None



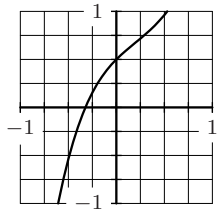
(i)



(ii)



(iii)



(iv)

2. Evaluate (find the sum of) the series $5 - \dfrac{10}{3} + \dfrac{20}{9} - \dfrac{40}{27} + \dfrac{80}{81} - \cdots$

3. Use the power series for e^x about $a = 0$ to find the power series for $z^2 e^{-z^3}$ about $a = 0$. Express your answer both in summation form, and by writing out the first four nonzero terms.

4. Determine whether the given series converges or diverges. **Justify your answers.**

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3 + 10}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4n^3 + 3n^2 + 5}$

5. Let $f(x) = \sqrt{x}$. Use **the definition** to calculate its Taylor polynomial of degree 3 at $a = 1$.

6. Both parts refer to the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2 \cdot 3^n}$.

   (a) Use the ratio test to find the radius of convergence.

   (b) Investigate the endpoint behavior, and determine the interval of convergence.

7. (a) Use the Comparison Test to determine whether or not

$$\sum_{n=0}^{\infty} \frac{n + 3^n}{2^n}$$

   converges.

(b) Use the Integral Test to determine whether or not

$$\sum_{k=0}^{\infty} \frac{k}{e^k}$$

converges.

(c) Use the Ratio Test to determine whether or not

$$\sum_{k=0}^{\infty} \frac{3^n}{n!}$$

converges.

8. Determine (with justification!) whether or not the following series converge:

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}, \qquad \sum_{n=0}^{\infty} (-1)^n \frac{n^2+4n+1}{3n^4+2n^2+10000}, \qquad \sum_{n=1}^{\infty} \frac{1}{n^{1.01}}.$$

9. Let $f(x) = \sqrt{1+x} = (1+x)^{1/2}$. Find the 4th degree Taylor polynomial of f centered at $a = 0$. Find a factorial expression for the general term of the Taylor series.

10. For each of the given power series, find the interval of convergence.

$$f(x) = \sum_{n=1}^{\infty} \frac{(2x)^n}{\sqrt{n}}, \qquad g(x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-5)^n}{n \cdot 3^n}.$$

(In particular, give the radius of convergence, and investigate convergence at the endpoints.)

11. The second degree Taylor polynomial of $f(x)$ at $a = 1$ is $p_2(x) = A + B(x-1) + C(x-1)^2$. What can you say about the signs of A, B, C if you know the graph of $f(x)$ is: