# MATH 132, Spring 2003

## Computer Lab #2

## Numerical and Symbolic Integration

## DUE DATE: Monday, Feb. 24, in class.

The goal for this lab project is for you to use MAPLE to numerically approximate integrals using left-hand and right-hand sums, the Midpoint rule, the Trapezoid rule and Simpson's rule. By comparing the errors of these methods with the actual values of the integrals, you can judge how effective each method is. You will also see how MAPLE can be used symbolically to evaluate integrals either using integration by parts or partial fractions. Much of the material covered here can be found in Chapter 7 of your text.

It is **required** that you work in a group of two or three people. Any help you receive from a source other than your lab partner(s) should be acknowledged in your report. For example, a textbook, web site, another student, etc. should all be appropriately referenced. Please turn in one report per group, listing the names of the groups members at the top of your report. Be sure to answer all questions carefully and neatly, writing in complete sentences. You should turn in your answers on separate sheets of paper from these instructions. (Keep this handout for later use.)

## Getting Started: Useful MAPLE Integration Commands

To execute some of the commands needed for this lab, you must load the student package which comes with the current version of MAPLE. Typing

```
with(student):
```

will load this package. Don't forget to load this package each time you work on the lab!

Recall that the left-hand and right-hand sums for a function $f(x)$ over an interval $[a, b]$ are methods for approximating the signed area under the graph of $f(x)$. We first divide the interval $[a, b]$ into $n$ equal pieces, (called a partition), yielding $n$ subintervals of width $\Delta x = (b-a)/n$. Then, we construct rectangles over each subinterval by choosing a representative height determined by the function. For a left-hand sum, we evaluate the function at the left endpoints of each subinterval, while for a right-hand sum, we use the right endpoints. The Midpoint rule is similarly defined by evaluating the function at the midpoint of each subinterval. The Trapezoid rule is obtained by approximating the area over each subdivision with a trapezoid. This is identical to taking the average of the left-hand and right-hand sums. Simpson's rule uses the Midpoint rule and Trapezoid rule to obtain an improved approximation.

MAPLE can compute these sums easily, using the commands

```
leftsum, rightsum, middlesum, trapezoid, simpson
```

For example, typing

```
evalf(middlesum(x^2,x=1..2,12));
```

gives the value of the Midpoint rule for $f(x) = x^2$ over $1 \leq x \leq 2$ with 12 rectangles (subdivisions). Typing

```
evalf(simpson(x^2,x=1..2,20));
```

gives the approximation of the integral using Simpson's rule, this time with 20 subdivisions. The `evalf` command is necessary to obtain a numerical answer. For calculating with other methods use the same exact syntax, just change the name of the method.

We can numerically or symbolically calculate a definite integral using the `int` command. MAPLE has its own built in numerical and symbolic integrators for obtaining answers. Some of these techniques we have learned in class. For example, to find the value of

$$\int_1^2 x^2\,dx$$

you can type `int(x^2,x=1..2);` If you do not obtain a numerical expression after using the `int` command, then try using an `evalf` command to tell MAPLE that you want a numerical answer outputted. For instance,

```
evalf(int(sin(x^2),x=0..Pi));
```

gives a numerical approximation to the integral of $\sin(x^2)$ from $x = 0$ to $x = \pi$.

To symbolically calculate the integral of $x^2$ you simply type `int(x^2,x)` . The first argument is the expression you are integrating while the second argument is the variable you wish to integrate with respect to. MAPLE can symbolically calculate many integrals but not all of them, as we shall see. Sometimes, MAPLE needs a little suggestion on how to proceed with the integral. For example, you can teach MAPLE how to do integration by parts. Suppose we wanted to do the integral

$$\int x\sin(x)\,dx$$

by parts. First define the integral by typing

```
A := Int(x*sin(x),x);
```

This assigns "A" to the integral you want to compute. We then use the `intparts` command which takes an integral as the first argument, and the expression you want $u$ to be as the second argument. So for example, if we wanted to let $u = x$ we type `intparts(A,x)` whereas if we wanted to let $u = \sin x$ we would type `intparts(A,sin(x))` . To evaluate the integral, we then use the `value` command:

```
value(intparts(A,x));
```

Try this to make sure you understand the syntax.

MAPLE also is capable of doing partial fractions (thank goodness). To find the partial fraction decomposition of

$$f(x) = \frac{3x^2 - 7}{(x+1)(x+2)(x-3)}$$

for example, we type

```
convert(f(x),parfrac,x)
```

where `f(x)` has been defined to be the function above. ( `f := x -> (3*x^2 - 7)` etc.) This gives the partial fraction decomposition of $f(x)$ after which it is easy for MAPLE to compute the integral.

## Exercises:

1. The following questions involve the integral

$$\int_{-3}^{3} 4x^2 - 3x + 5 \, dx$$

   (a) Find the exact value of the integral by hand (without any technology). You can check your answer using MAPLE with the `int` command.

   (b) Use Simpson's rule to numerically approximate the integral using $n = 2, 10, 50, 100, 500$ subdivisions. What do you notice about the results you obtain? Explain why this happens.

   (c) Compute the values of LEFT(500), RIGHT(500), TRAP(500) and MID(500) for this integral using the commands described above. Make a table of values along with the absolute value of the error for each method. What is the ratio of the errors between the Midpoint rule and the Trapezoid rule?

   (d) Are there any functions for which the Left-hand and Right-hand sums give the **exact** value of the integral, regardless of the number of subdivisions? Explain. What about for the Midpoint and Trapezoid Rules? Test your conjecture by trying some examples with MAPLE.

   (e) *Extra Credit:* Using Problem #11 of Section 7.6 of the text, prove that Simpson's Rule gives the **exact** value to the integral of any quadratic function.

2. The following questions involve the integral

$$\int_{0}^{2} \sin(2x^2 + 1) \, dx$$

   (a) Use the `int` and `evalf` commands to numerically approximate the integral using MAPLE. Note that this integral is impossible to do other than through numerical techniques, so this answer is the best we can hope for.

   (b) Compute the left-hand and right-hand sums for the integral using $n = 2, 10, 100, 500$ subdivisions. At each stage compute the absolute value of the error with the value obtained in part (a).

   (c) Compute the integral using the Trapezoid and Midpoint rules using $n = 2, 10, 100, 500$ subdivisions. At each stage compute the absolute value of the error with the value obtained in part (a).

   (d) Compute the integral using Simpson's rule using $n = 2, 10, 100, 500$ subdivisions. At each stage compute the absolute value of the error with the value obtained in part (a).

   (e) Draw some conclusions about each method based on parts (b), (c) and (d). List the methods based on their effectiveness. How does the error of each method improve as the number of subdivisions increases? More specifically, how many decimal places of accuracy do you gain at each stage, for each method? Which method is improving at the fastest rate?

3. Using MAPLE, try and compute the integral

$$\int x \ln(x + \sqrt{1 + x^2})\, dx$$

using the `int` command. What happens? Now try doing integration by parts with $u = x$ using the `intparts` and `value` commands. Is this an improvement? Finally, try letting $u = \ln(x + \sqrt{1 + x^2})$ and use MAPLE to do integration by parts. This should give the integral. Check your answer by hand just to make sure. (Ok, this is a tedious calculation but it is important to check the computer's output when possible.)

4. Use MAPLE to find the partial fraction expansion for $\dfrac{x^5 + 4x - 1}{(x-1)^4(x^2+3)}$ and then find the integral

$$\int \frac{x^5 + 4x - 1}{(x-1)^4(x^2+3)}\, dx.$$

Using the partial fraction expansion, show where each term in the evaluated integral comes from.