MATH 131, Sections 02 and 04, Fall 2005

Calculus Lab #3

Hyperbolic Functions

DUE DATE: Monday, November 7th, Start of class

The goal for this project is to introduce hyperbolic functions using analytic and graphical techniques. Specifically, we will explore the similarities and differences between the hyperbolic functions and their trigonometric counterparts. Most of the problems can be done using pencil and paper, although you can use Maple or a calculator for some of the computations.

It is **required** that you work in a group of two or three people. Any help you receive from a source other than your lab partner(s) should be acknowledged in your report. For example, a textbook, web site, another student, etc. should all be appropriately referenced. Please turn in one report per group, listing the names of the groups members at the top of your report.

Your lab should consist of coherent and sufficiently detailed answers to the questions below. Be sure to answer all questions carefully and neatly, writing in complete sentences. As always, SHOW YOUR WORK. You do **NOT** need to type up your answers for this lab.

Hyperbolic Functions

There are two functions involving e^x and e^{-x} that are used so frequently in applications of Calculus that they are given their own names. For example, these functions appear frequently in a subject called *Partial Differential Equations*, where phenomenon involving waves and diffusion of heat are studied. These functions are the **hyperbolic cosine** function $\cosh x$ and the **hyperbolic sine** function $\sinh x$ defined by

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \text{and} \qquad \sinh x = \frac{e^x - e^{-x}}{2}.$$

Note that the only difference between $\cosh x$ and $\sinh x$ is the negative sign in the numerators. However, this is a big difference!

Figure 1 shows the graphs of $\cosh x$ and $\sinh x$. The names of these functions suggest a comparison with their trigonometric counterparts $\cos x$ and $\sin x$. For example, $\cosh x$ and $\cos x$ are both even functions and $\cosh 0 = \cos 0 = 1$. Similarly, $\sinh x$ and $\sin x$ are both odd functions and $\sinh 0 = \sin 0 = 0$.

Problems

1. Verify that $f(x) = \cosh x$ is an even function by showing that $f(-x) = f(x)$ (the definition of an even function.)

2. Verify that $g(x) = \sinh x$ is an odd function by showing that $g(-x) = -g(x)$ (the definition of an odd function.)

3. Recall that $\cos^2 x + \sin^2 x = 1$ for any angle x. Is the corresponding formula for $\cosh x$ and $\sinh x$ true? In other words, what is $\cosh^2 x + \sinh^2 x$?

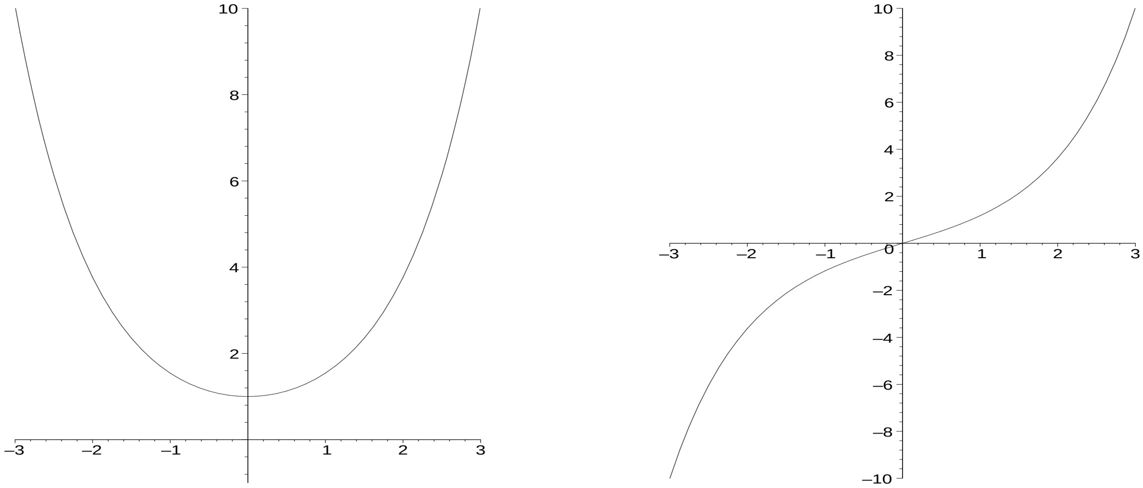

Figure 1: The graphs of $\cosh x$ (left) and $\sinh x$ (right).

4. Verify that

$$\cosh^2 x - \sinh^2 x \ = \ 1.$$

This reveals where the adjective "hyperbolic" comes from. If we let $x = \cosh t$ and $y = \sinh t$ with t a parameter, then x and y trace out the **hyperbola** $x^2 - y^2 = 1$ in the xy-plane. We will discuss parameterized curves in more detail in Section 4.8.

5. Compute and simplify the derivatives of $\cosh x$ and $\sinh x$. How do these relate to the derivatives of $\cos x$ and $\sin x$? Explain. What is the 2005th derivative of $\cosh x$?

We can define other hyperbolic functions exactly as we do in trigonometry. For example, $\tanh x$ and sech x, **hyperbolic tangent** and **secant**, respectively, are defined as

$$\tanh x \ = \ \frac{\sinh x}{\cosh x} \qquad \text{and} \qquad \operatorname{sech} x = \frac{1}{\cosh x}.$$

Our next goal is to compare the properties of $\tanh x$ with its trigonometric counterpart $\tan x$.

6. **a)** Compute $\tanh 0$.
   b) Compute and simplify the derivative of $\tanh x$.
   c) On what intervals is $\tanh x$ increasing? On what intervals is $\tanh x$ decreasing?
   d) Compute and simplify the second derivative of $\tanh x$.
   e) On what intervals is $\tanh x$ concave up? On what intervals is $\tanh x$ concave down?
   f) Compute the limits

$$\lim_{x\to\infty} \tanh x \quad \text{and} \quad \lim_{x\to-\infty} \tanh x.$$

   g) Using all of the information obtained above, draw the graph of $\tanh x$. You may use Maple to check your result, but you should be able to draw the graph without resorting to a computer (or calculator). In what ways is the $\tanh x$ function similar to and different from $\tan x$? Explain.