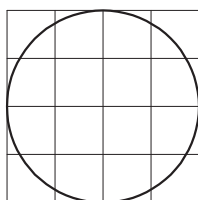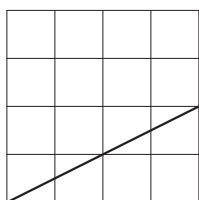**College of the Holy Cross, Fall Semester, 2005**
Math 131, Practice Midterm 3

1. Match each parametric curve with its graph. (Each graph shows the square $-1 \le x \le 1$, $-1 \le y \le 1$.)
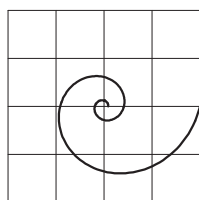
   (a) $x(t) = -1 + 2t$, $y(t) = 1 - 2t$, $0 \le t \le 1$ ◯ I ◯ II ◯ III ◯ IV ⊗ V

   (b) $x(t) = \cos(2t)$, $y(t) = \sin(2t)$, $0 \le t \le \pi$ ⊗ I ◯ II ◯ III ◯ IV ◯ V

   (c) $x(t) = \cos t$, $y(t) = \frac{1}{2}\sin t$, $0 \le t \le 2\pi$ ◯ I ◯ II ◯ III ⊗ IV ◯ V

   (d) $x(t) = 1 - 2t$, $y(t) = -t$, $0 \le t \le 1$ ◯ I ⊗ II ◯ III ◯ IV ◯ V
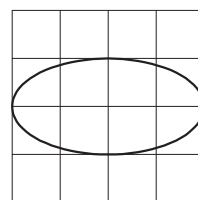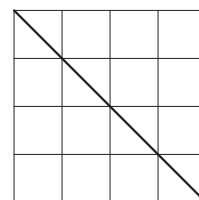

     I   II   III   IV   V

2. Compute the indicated limits. Show all work for full credit.

   (a) $\displaystyle\lim_{x \to 1} \frac{\cos\left(\frac{\pi}{2}x\right)}{x - 1}$

   (b) $\displaystyle\lim_{x \to 0} \frac{x^2 + 1}{x^2 + 3}$

   (c) $\displaystyle\lim_{x \to \infty} \frac{x \ln x}{x^{1.01}}$

3. Each part refers to the function $f(x) = 2x^3 - 6x^2$.

   (a) Find and classify (local min/max, or neither) the critical points of f.

   (b) Find the maximum and minimum values of $f(x)$ if $-\frac{3}{2} \le x \le \frac{5}{2}$.

   (c) In the grid provided, sketch the graph $y = f(x)$ for $-1 \le x \le 3$. (Note that axis labels are provided.) For full credit, clearly indicate the critical and inflection point(s) in this interval, and label each such point with *both coordinates*.
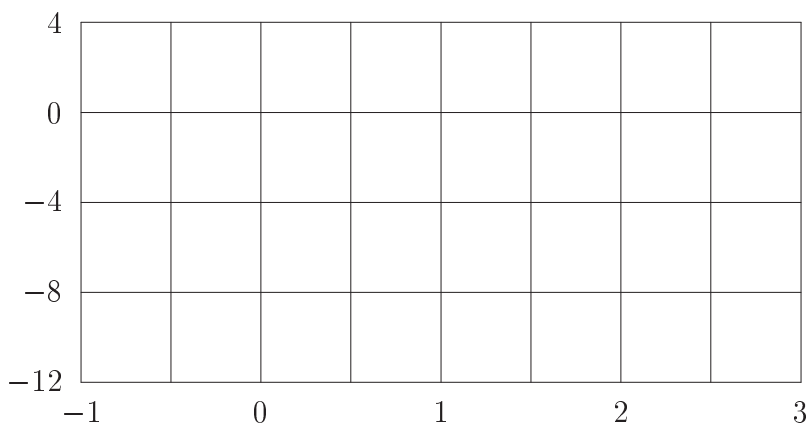
4. Crusader Movie Rentals finds that they can rent 160 movies per night at \$1 per movie. For every dollar that the rental fee increases, 40 fewer movies are rented. What price should be charged to maximize the revenue (total rental income)?

5. Find the equation of the line tangent to the curve $xy - 2x - 3y + 1 = 0$ at the point $(-2, 1)$.

6. A boat is drawn into a dock by a rope over a small pulley. The pulley is five feet higher than the bow of the boat (see figure). Let ℓ be the length of rope, x the distance from the boat to the dock.
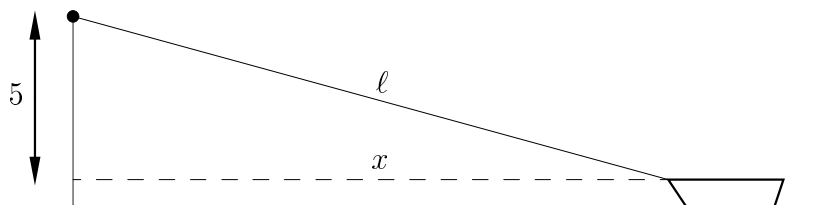
(a) Find an equation relating ℓ and x, and determine x when $\ell = 13$.

(b) Suppose the rope is drawn in at 3 ft/sec. How fast is the boat moving when the length of the rope is 13 feet?