

College of the Holy Cross, Fall Semester, 2005
Math 131, Practice Final

1. Circle the number corresponding to the graph of each function. Each square in the figures is 1 unit by 1 unit, and the bold lines are the axes.

(a) $\frac{1}{2}x - 2$ I II III IV V VI VII VIII IX

(b) $\sin(4x)$ I II III IV V VI VII VIII IX

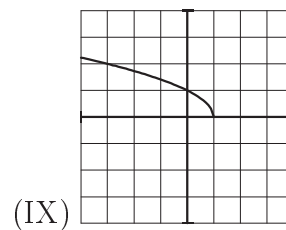
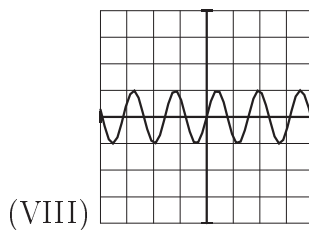
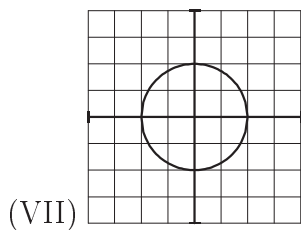
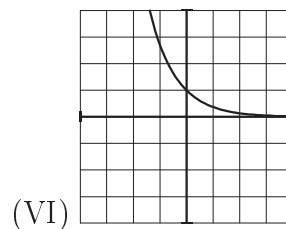
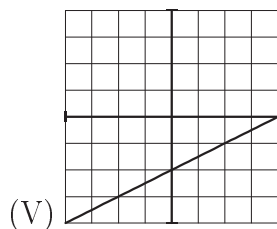
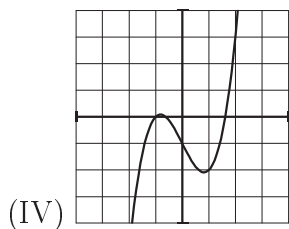
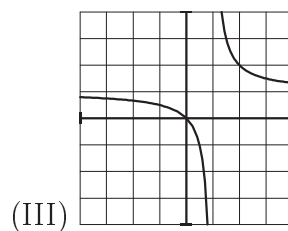
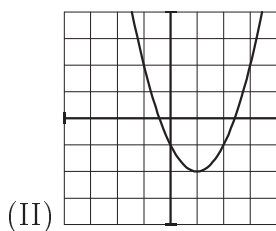
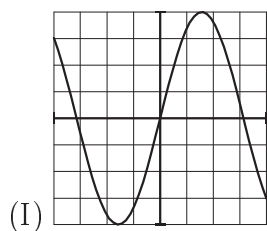
(c) $x^3 - 2x - 1$ I II III IV V VI VII VIII IX

(d) $\frac{x}{x-1}$ I II III IV V VI VII VIII IX

(e) e^{-x} I II III IV V VI VII VIII IX

(f) $x^2 - 2x - 1$ I II III IV V VI VII VIII IX

(g) $4\sin(x)$ I II III IV V VI VII VIII IX



2. Compute the following limits.

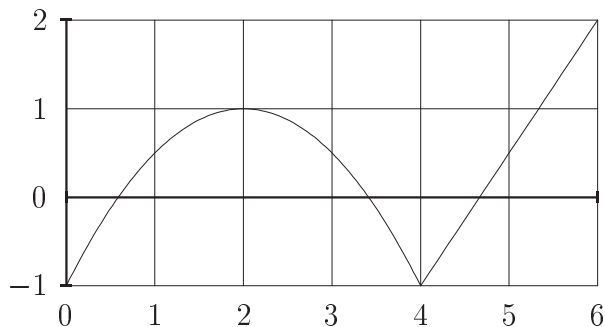
(a) $\lim_{x \rightarrow 0} \frac{x^2 - 9}{3 - x}$

(b) $\lim_{x \rightarrow 1} \frac{2^x - 2}{\ln(x)}$

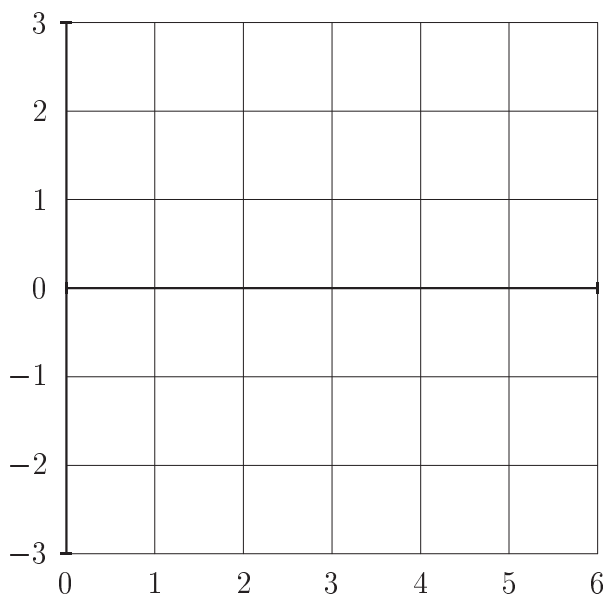
(c) $\lim_{x \rightarrow \infty} \frac{4x^2 + 3e^{-x}}{5x^2 + 7e^{-x}}$

3. The graph of the function $f(x)$ is shown below. On the axes provided, sketch the graph of $f'(x)$. Be sure to label the points where f' is zero or undefined.

Graph of f :



Graph of f' :



4. (a) Suppose $f(2) = 3$ and $f'(2) = -5$. Use the linear approximation of f at $x = 2$ to estimate the value of $f(1.97)$.
- (b) Suppose as in part (a) that $f(2) = 3$ and $f'(2) = -5$. Also suppose $g(3) = -1$ and $g'(3) = 7$. Let $h(x) = g(f(x))$. Find $h'(2)$.

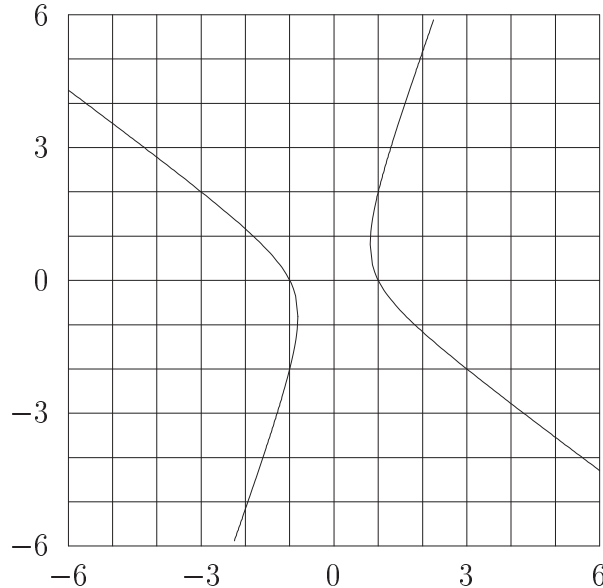
5. Compute the derivative of each function.

(a) $f(x) = \frac{x^3}{2x^2 + 1}$

(b) $g(x) = x^2 \cos(3x)$

(c) $h(x) = \ln(1 + e^{\sqrt{x}})$

6. The hyperbola $2x^2 + 2xy - y^2 = 2$ is shown below.



(a) Compute $\frac{dy}{dx}$ in terms of x and y .

(b) Find the equation of the tangent line to the curve at the point $(1, 2)$ and sketch the tangent line on the figure above.

7. Suppose the position of an object is described by the parametric curve $x = t^3 - t^2$, $y = 3t^3 - 4t$, where t is measured in seconds.

(a) Find the location of the object at time 2 seconds. Find the equation of the tangent line to the curve at this point.

(b) At what time is the speed of the object zero?

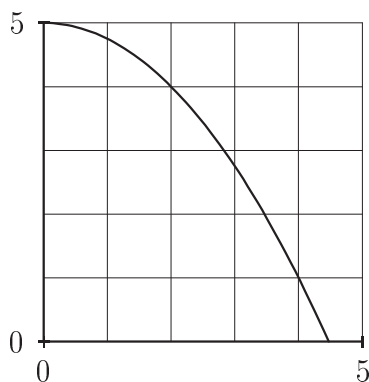
8. Let $f(x) = x^3 e^x$.

(a) Find and classify (local min/max, or neither) the critical points of f .

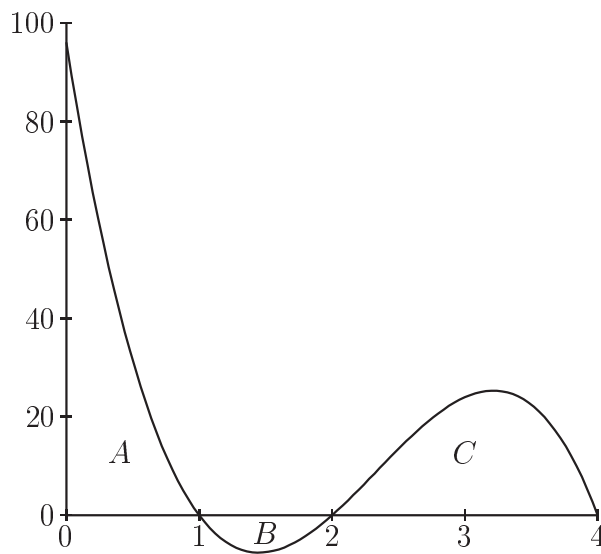
(b) Find the inflection points of f .

9. You have \$3 with which to construct a box with a square base. The material for the top and bottom costs 2 cents per square inch, while the material for the sides costs 1 cent per square inch. What dimensions maximize the volume of the box? *Hint:* First write the cost of the box and the volume of the box in terms of the dimensions of the box.

10. Suppose you are making a snow-person, and you begin by rolling a snowball in the snow. Assume the snowball always remains in the shape of a sphere. If the radius of the snowball increases at a constant rate of 2 inches per second, how fast is the volume of the snowball increasing when the radius is 6 inches? Give the units of your answer. The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.
11. The graph of $f(x) = 5 - \frac{1}{4}x^2$ is shown below.



- (a) Compute the left and right hand sums for f over the interval $1 \leq x \leq 4$ using $n = 3$ subintervals. On the figure above sketch the rectangles which represent the terms in the left and right hand sums.
- (b) Which sum is a better estimate of the exact area under the graph of f over $1 \leq x \leq 4$? Explain.
12. The graph of $f(x)$ is shown below. The areas of the regions shown are $A = 37$, $B = 5$ and $C = 32$.



- (a) Evaluate $\int_0^4 f(x) dx$
- (b) Find the average value of f over the interval $1 \leq x \leq 4$.

13. Suppose the rate of growth of a population of bats living under a bridge, in thousands of bats per year, is

$$f(t) = 2 + \frac{t}{3}$$

where t is measured in years since 1990. Suppose there were 40 thousand bats in 1990.

- (a) Write out an integral that represents the change in the bat population from 1990 to 2005.
- (b) Compute the integral exactly (use the graph of f) and determine what the population will be in 2005.