

MATH 131, Fall 2002

Computer Lab #4

Lissajous Figures

DUE DATE: Monday, Nov. 18th, in class.

The goal for this lab project is to further your understanding of parametric equations by exploring a special type of curve called a **Lissajous figure**. Using MAPLE, you will draw several different figures by varying certain **parameters** in the defining equations. The aim is for you to use MAPLE to make some predictions and draw some conclusions about the effect these parameters have on the corresponding Lissajous figure. Through this exploration you should develop a better understanding of parameterized curves, in particular, the connection between the two component functions and the path traced out by the particle in the xy -plane.

It is **required** that you work in a group of two or three people. Any help you receive from a source other than your lab partner(s) should be acknowledged in your report. For example, a textbook, web site, another student, etc. should all be appropriately referenced. Please turn in one report per group, listing the names of the groups members at the top of your report. Be sure to answer all questions carefully and neatly, writing in complete sentences. You should turn in your answers on separate sheets of paper. There are three plots via MAPLE which need to be turned in for this project.

Lissajous Figures

The French physicist Jules Antoine Lissajous (1822-1880) (pronounced Lee-suh-zhoo) was interested in studying waves and vibrations. Attaching mirrors to two tuning forks which were vibrating at *different* frequencies, he was able to project light off of the mirrors and onto one screen to create what is today called a **Lissajous figure**. His setup was similar to the modern device used to project laser light shows. Lissajous figures were used in the old days to determine the frequencies of sounds or radio signals. They found their way into popular culture in many sci-fi movies and TV shows, including the opening footage for *The Outer Limits* TV series. (“Do not attempt to adjust your picture — *we* are controlling the transmission.”) [1,2].

To create Lissajous figures, we will use the parameterization

$$\begin{aligned}x &= \cos\left(\frac{2\pi}{a}t\right) \\y &= \sin\left(\frac{2\pi}{b}t\right)\end{aligned}$$

where a, b are always taken to be positive real numbers. The equations are chosen so that the period of the horizontal component x is a and the period of the vertical component y is b . To begin, define the parameterization by typing

```
r := t -> (cos(2*Pi*t/a), sin(2*Pi*t/b));
```

exactly into Maple. Each time you draw a Lissajous figure, you will save a lot of time if you redefine a and b as you go. For example, assuming you have defined r correctly, the commands

```
a := 3: b := 1:
plot([r(t), t=0..2]);
```

should draw the Lissajous figure with $a = 3$ and $b = 1$ over the time interval $0 \leq t \leq 2$. Try plotting a few sample figures before answering the questions below. The website

<http://surendranath.tripod.com/Lissajous/Lissajous.html>

contains a nice Java applet which draws Lissajous figures, although the parameterizations there are *different* from the ones you will be using here. This site is recommended if you are having trouble understanding why the figures look the way they do.

1. Begin by setting $a = b = 1$. What figure do you obtain? Using the parameterization above, what is the **period** of this figure? By period, we mean how long does it take until the trajectory returns to its starting position and direction. What figure will you obtain whenever $a = b$? Why? No graphs need to be turned in for this part.
2. Use Maple to plot the Lissajous figure for $a = 2$ and $b = 1$. Print out your plot and label at least 5 points on the figure with their corresponding times. What is the period of this curve? It may be useful to you to plot the component functions x and y separately when trying to label different times on your Lissajous figure.
3. Keeping b fixed at 1 while varying a to 3, 4, 5, ..., use Maple to plot the corresponding Lissajous curves. What changes do you see and why do they occur? What is the period when $b = 1$ and $a = 3$? What is the period when $b = 1$ and $a = k$, for any positive integer k ? Print out one particular Lissajous curve from this example and turn it in, labeling at least 5 points on the figure with their corresponding times.
4. Use Maple to plot the Lissajous figure for $a = 1$ and $b = 2$. How is this different from the plot in Question 2? Explain. Would you say this curve is periodic? In other words, does it ever return to its starting point and then retrace its path again? What is the period of this curve? What shape is this curve? Can you find an equation between x and y which describes this curve? Print out your plot and label at least 5 points on the figure with their corresponding times.
5. Keeping a fixed at 1 while varying b to 3, 4, 5, ..., use Maple to plot the corresponding Lissajous curves. What changes do you see and why do they occur? What is the period when $a = 1$ and $b = 3$? What is the period when $a = 1$ and $b = k$, for any positive integer k ? Do all the Lissajous curves obtained in this part close up on themselves (complete a cycle)? Which ones close up and which ones don't? Can you explain why?
6. By experimenting with different integer values of a and b other than the case $a = b$, what can you conclude about the corresponding Lissajous figure? What role do a and b play? Try fixing one of a or b constant and varying the other to look for patterns in the figures. (Much of mathematical research begins with looking for patterns!) What is the period of the Lissajous figure if $a = 5$ and $b = 3$? What is the period of the Lissajous figure if $a = 4$ and $b = 6$? Find an expression for the period given any two integer values for a and b .
7. The two Lissajous figures in Figure 1 were created using the $r(t)$ parameterization given above. Find the integer values of a and b used in each case. Explain how you obtained your answers. Note that the figure on the left closes up on itself while the figure on the right does not.
8. All the examples above have used a and b as integers. What happens if you try $a = 1$ and $b = \sqrt{2}$? The syntax for the square root is `sqrt(2)`. Is the curve periodic? What happens as you plot the curve over longer and longer time intervals? Explain.

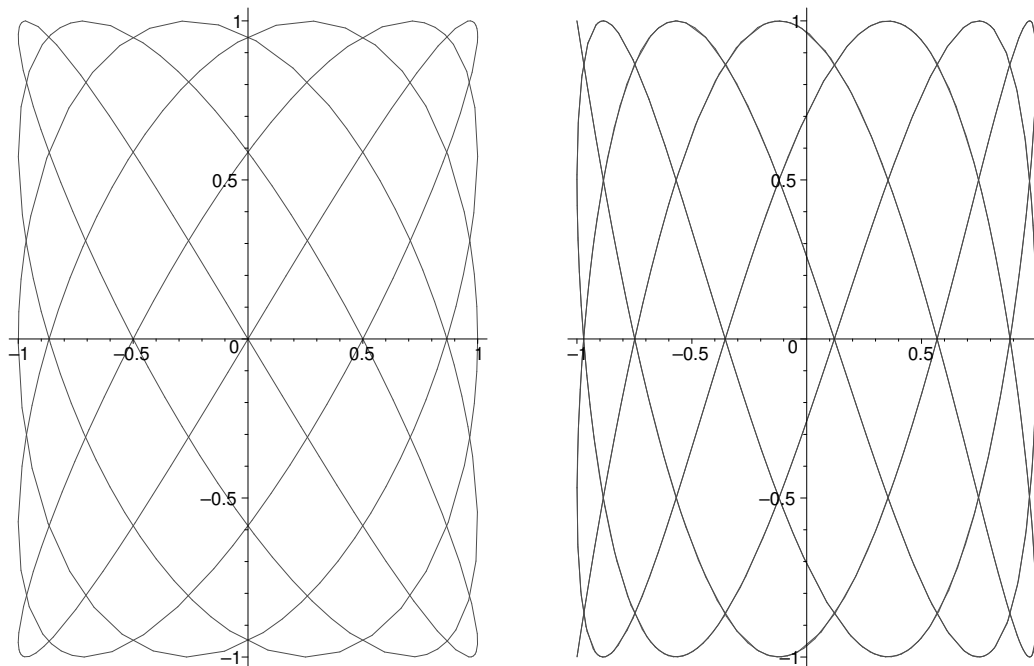


Figure 1: Two typical Lissajous figures. Can you find the particular values of a and b used in each case?

References

1. Hobbs Ed., *Lissajous Lab*, website: “<http://www.math.com/students/wonders/lissajous/lissajous.html>” maintained by Math.com.
2. MacTutor History of Mathematics archive, *Jules Antoine Lissajous*, website: “<http://www-groups.dcs.st-andrews.ac.uk/history/Mathematicians/Lissajous.html>” maintained by the School of Mathematics and Statistics, University of St. Andrews, Scotland.