MATH 131, Fall 2002

Computer Lab #2

Understanding Rational Functions

DUE DATE: Monday, Sept. 30th, in class.

The goal for this lab project is to develop a solid understanding of rational functions. We will use MAPLE to visualize these functions, although it may not be obvious what size window is useful for viewing a given function. Many of the ideas on rational functions from Section 1.6 of the class text are focused on, especially vertical and horizontal asymptotes. The concept of the limit of a function is approached in this project as well.

It is **required** that you work in a group of two or three people. Any help you receive from a source other than your lab partner(s) should be acknowledged in your report. For example, a textbook, web site, another student, etc. should all be appropriately referenced. Please turn in one report per group, listing the names of the groups members at the top of your report. Be sure to answer all questions carefully and neatly, writing in complete sentences. You should turn in your answers on seperate sheets of paper.

1. Use MAPLE to define the polynomial $p(x) = x^3 - 5x + 2$ and then plot it over the domain $-4 \leq x \leq 4$. The commands for this are listed below if you have forgotten them. Be sure to type them in exactly as below.

```
p := x -> x^3 - 5*x + 2;
plot(p, -4..4);
```

   (a) What is the **degree** of $p(x)$? What is the **leading coefficient** of $p(x)$?

   (b) Find all the **roots** (zeroes) of $p(x)$. Estimate them as best as you can using MAPLE. Can you find their *exact* values?

   (c) What happens to the values of $p(x)$ as x approaches infinity? What happens as x approaches negative infinity? In other words, as the values of x become larger and larger, does $p(x)$ approach infinity, does $p(x)$ approach negative infinity, does $p(x)$ approach a finite value or does none of the above happen? You may use your graph by adjusting the window range. In mathematical terms, you are answering the questions

$$\lim_{x\to\infty} p(x) = ? \qquad \text{and} \qquad \lim_{x\to-\infty} p(x) = ?$$

2. Keep your definition of $p(x)$. Now plot the function $q(x) = -3x^3 + 4x^2$ over a suitable window of x and y values. Answer questions (a), (b) and (c) from Problem 1 above, but applied to $q(x)$.

3. Now define the **rational function** $r(x) = p(x)/q(x)$. It is called a rational function because it is the ratio of two polynomials. The easiest way to define r is to type `r := x -> p(x)/q(x);`

   (a) Plot $r(x)$ on a window of x and y values which shows the whole behavior of the function. Turn in your graph of $r(x)$. (Use Print from the File menu. You will need to log on with your Novell account in order to use the printer in the lab.)

(b) Find all the **roots** (zeroes) of $r(x)$. Estimate them as best as you can using MAPLE. Can you find their *exact* values? How does this relate to Problem 1? Explain.

(c) What new features appear in the graph of $r(x)$ which did not arise in the graphs of p and q? Explain why these appear, relating your answer to Problem 2.

(d) Where is the function $r(x)$ positive? Where is it negative? Give specific answers. Explain how your answers relate to the values of $p(x)$ and $q(x)$.

(e) What happens to the values of $r(x)$ as x approaches infinity? What happens as x approaches negative infinity? Use your graph to answer this by adjusting the window range. Confirm your answer by calculating the actual limits (as done in class).

4. For each of the following rational functions, use MAPLE to plot the function and locate all vertical and horizontal asymptotes (if they exist). Be sure to explain your answers carefully, with justification as needed. You may wish to approach the problem as above, by defining p and q in each case. You do NOT need to turn in any graphs for this question.

(a) $\dfrac{4x^4 - x^3 - 10x^2 + 1}{x^4 + 2x^2 - 1}$

(b) $\dfrac{x^5 - 3x^3 + 2x}{x^4 - x^3 - 7x^2}$

(c) $\dfrac{x^6 + 10x^4 - 10x^3 + 5}{x^7 - 1}$