MATH 126 Sample Final Exam Questions

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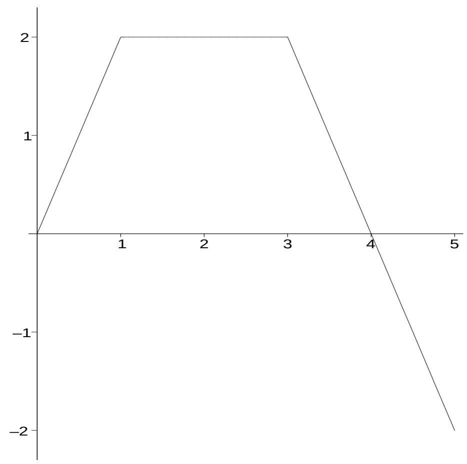
Below are some **sample** final exam questions. Collectively, these are not intended to represent an actual exam nor do they completely cover all the material that could be asked on the exam.

1. Define $F(x) = \int_0^x f(t)\,dt$ for $0 \leq x \leq 5$, where the graph of $f(t)$ is given below.



(a) Find $F(0)$ and $F(5)$.

(b) Find $F'(1)$ if it exists. If it does not exist, explain why.

(c) Find $F''(1)$ if it exists. If it does not exist, explain why.

(d) Find the intervals on which F(x) is increasing and decreasing.

(e) Find the intervals on which F(x) is concave up and concave down.

(f) Sketch a graph of $F(x)$ over the interval $0 \leq x \leq 5$.

2. Evaluate the following integrals.

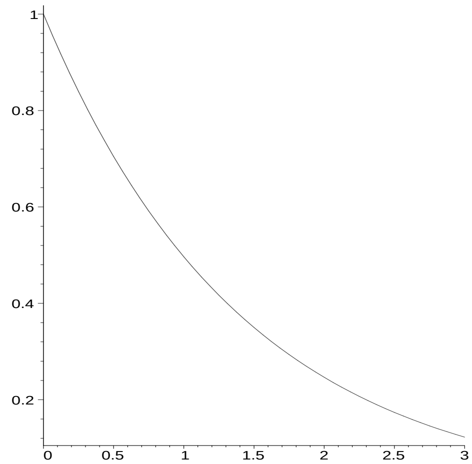
(a) $\displaystyle\int \sqrt{2x+1} + \sin(3x)\ dx$

(b) $\displaystyle\int \sec^2(2\theta)\, e^{\tan(2\theta)}\ d\theta$

(c) $\displaystyle\int x^6 \ln x\ dx$

(d) $\displaystyle\int \frac{z^2}{\sqrt{1-z^2}}\ dz$ *Hint:* Use the substitution $z = \sin\theta$ and then use $\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$.

(e) $\displaystyle\int \frac{6}{x(x-1)(x+1)}\ dx$



3. Consider the function $g(x)$ whose graph is shown above:

   The following numbers are the left, right, trapezoid and midpoint approximations to $\int_0^3 g(x)\,dx$, each with $n = 50$ subdivisions.

   (I) 1.227491645 (II) 1.253541536 (III) 1.253817953 (IV) 1.280144260

   (a) Match each value with one of the four approximations, explaining your choices.

   (b) Use Simpson's rule to approximate the value of the integral.

4. Let R be the region in the first quadrant bounded by $y = \sqrt{x}$ and $y = x^2$.

   (a) Find the area of the region R.

   (b) Find the volume of the solid of revolution obtained by rotating R about the x-axis.

   (c) Find the volume of the solid of revolution obtained by rotating R about the y-axis.

5. The demand function for a given product is $p = 66 + \frac{384}{x+2}$ and the supply curve is $s = 4x + 58$.

   (a) Find the equilibrium price $\bar{p}$ and the equilibrium quantity $\bar{x}$.

   (b) Find the consumer surplus and the producer surplus.

6. Find the solution to the given initial-value problems:

   (a) $\dfrac{dy}{dt} = \dfrac{y}{1+t^2}, \quad y(0) = 3.$

   (b) $\dfrac{dy}{dx} = y^{2010} - 1, \quad y(0) = 1.$

7. Consider the initial-value problem

$$\frac{dy}{dx} = 2(x+1)y^2, \quad y(0) = 1/2.$$

(a) Using Euler's method with a step-size of $\Delta x = 0.2$, approximate the value of the solution when $x = 1$. In other words, approximate $y(1)$ where $y(x)$ is the solution to the given ODE.

(b) Solve the ODE with the given initial condition.

(c) Using your answer to part **(b)**, calculate $y(1)$. What is the error in your approximation? (Any ideas to why it is so far off?)

8. Hermione Granger, who scored an Outstanding on her O.W.L. exam in the subject of Arithmancy, has to solve a murder that has dreadfully occurred right in the common room of her very own house, Gryffindor. The body was discovered at 7:00 am with a temperature of 85.2°F. At 9:00 am the temperature of the body has dropped to 81.5°F. Using magic, the temperature of the common room is always held constant at 68°F.

   Hermione has cleverly deduced that the killer is one of three Slytherin students: Draco Malfoy, Pansy Parkinson, or Vincent Crabbe. Draco has an alibi from Midnight until 4:30 am, Pansy has an alibi from 5:00 - 11:30 pm and 2:00 - 6:00 am, and Crabbe has an alibi from 7:00 pm until 2:00 am. Assuming that the temperature of a healthy human body is 98.6°F, use Newton's Law of Cooling to find the time of death to the nearest minute. If Hermione is correct and one of her three suspects is the murderer, who must be guilty? (Final Jeopardy Question)

9. Sequences and Series:

   (a) Find a formula for the general term a_n for the sequence
   $$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, + - \dots$$

   (b) Does the sequence given by $a_n = \frac{1-n^3}{4+3n^3}$ converge or diverge? If it converges, find the limit.

   (c) Find the sum of the given geometric series: $18 - 6 + 2 - 2/3 + 2/9 - + \cdots$

   (d) Does the given infinite series converge or diverge? If it converges, find the limit.
   $$\sum_{n=1}^{\infty} \frac{3^n}{\pi^{n+1}}$$

10. Some conceptual questions:

   (a) Derive the formula for the volume of a sphere of radius r by rotating the top half of the circle $x^2 + y^2 = r^2$ about the x-axis.

   (b) Find the average value of the function $x \sin x$ over the interval $0 \leq x \leq \pi$.

   (c) Suppose that $f(x)$ is a piecewise function defined as follows:
   $$f(x) = \begin{cases} 0 & \text{if } x < 1 \\ \dfrac{k}{x^3} & \text{if } x \geq 1 \end{cases}$$
   Find the value of k which makes f a probability density function (pdf).

   (d) For what values of k does $y = e^{kt}$ satisfy the differential equation $y'' - 3y' - 4y = 0$?

   (e) Give an example of an ODE that has an **infinite** number of equilibrium points.