MATH 125-03/04 Quiz #8 Solutions

November 17, 2009 Prof. Gareth Roberts

Suppose that $x(t)$ and $y(t)$ are two **positive** quantities that depend on time. If x and y are related by the equation $x^2 + xy + y^2 = 19$, and if $\frac{dx}{dt} = \frac{1}{4}$, find $\frac{dy}{dt}$ when $x = 3$. (10 pts.)

Solution: Writing $x = x(t)$ and $y = y(t)$, we have $(x(t))^2 + x(t) \cdot y(t) + (y(t))^2 = 19$. Differentiating this equation with respect to t using the chain rule and product rule gives

$$2x(t) \cdot \frac{dx}{dt} \; + \; \frac{dx}{dt} \cdot y(t) \; + \; \frac{dy}{dt} \cdot x(t) \; + \; 2y(t) \cdot \frac{dy}{dt} \; = \; 0$$

which simplifies to

$$(2x + y)\frac{dx}{dt} + (x + 2y)\frac{dy}{dt} \; = \; 0. \tag{1}$$

We have values for x and dx/dt. To find y substitute $x = 3$ into the original equation. This gives the quadratic equation $9 + 3y + y^2 = 19$ or $y^2 + 3y - 10 = 0$. Factoring, we have $(y + 5)(y - 2) = 0$ which implies that $y = 2$ or $y = -5$. Since y is assumed to be positive, we have that $y = 2$. Substituting this value and those given in the problem into equation (1) yields

$$(2 \cdot 3 + 2) \cdot \frac{1}{4} + (3 + 2 \cdot 2)\frac{dy}{dt} \; = \; 0$$

and thus, $dy/dt = -2/7$.