MATH 125 Sample Final Exam Questions

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1. Circle the number corresponding to the graph of each function. Each square in the figures is 1 unit by 1 unit, and the bold lines are the axes.

   (a) $\frac{1}{2}x - 2$ I   II   III   IV   V   VI   VII   VIII   IX

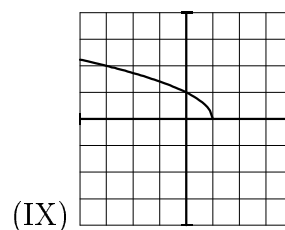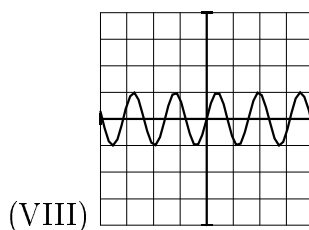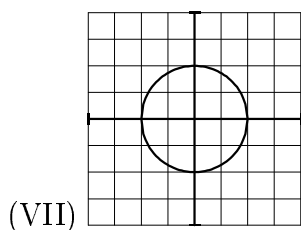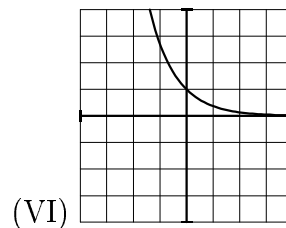   (b) $\sin(4x)$ I   II   III   IV   V   VI   VII   VIII   IX

   (c) $x^3 - 2x - 1$ I   II   III   IV   V   VI   VII   VIII   IX

   (d) $\dfrac{x}{x - 1}$ I   II   III   IV   V   VI   VII   VIII   IX

   (e) e^{-x} I   II   III   IV   V   VI   VII   VIII   IX

   (f) $x^2 - 2x - 1$ I   II   III   IV   V   VI   VII   VIII   IX

   (g) $4\sin(x)$ I   II   III   IV   V   VI   VII   VIII   IX
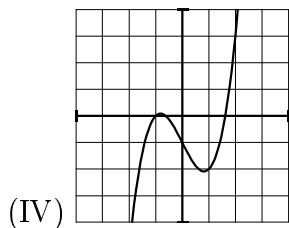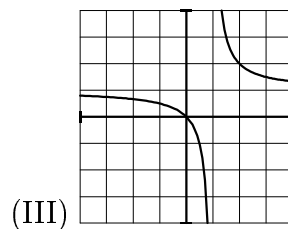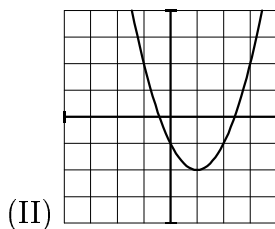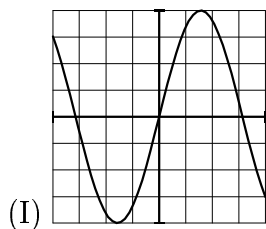
(I)   (II)   (III)

(IV)   (V)   (VI)

(VII)   (VIII)   (IX)
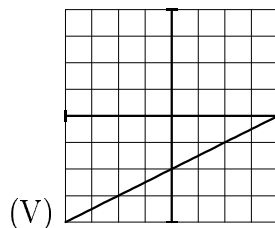
2. Find the equation of the tangent line to the curve defined by $e^{xy} + x^2 + y^2 = 10$ at the point $(0, 3)$.

3. Suppose that $D(t)$ is the depth (in inches) of snow on your lawn t days after January 1st. Provide an interpretation in words of the equation $D^{-1}(5) = 3$?

4. Compute the derivative of each function. Simplify your answer as best as possible.

   (a) $f(x) = x^2 e^{\tan x}$

   (b) $g(t) = \dfrac{1}{\sqrt{t^4 + 4t^3}}$

   (c) $h(x) = \cos(2^x)$

   (d) $y = \tan^{-1}(\ln(5x))$;

5. Consider the graph of $f(x)$ and $g(x)$ shown below.
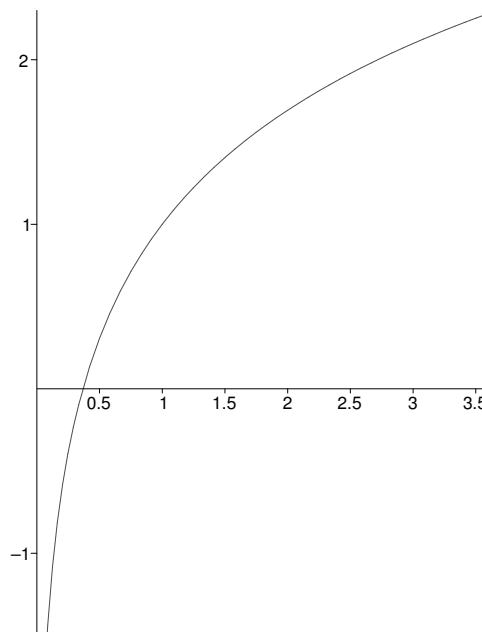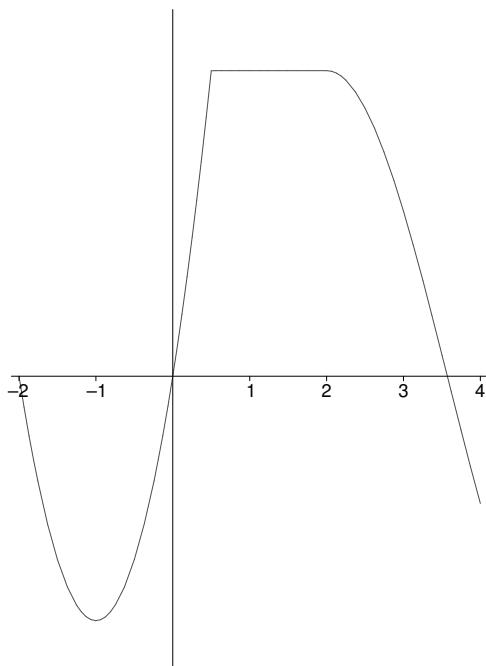
   (a) At what points (if any) is $f(x)$ **NOT** differentiable?

   (b) Sketch the graph of $f'(x)$.

   (c) Sketch the graph of $g'(x)$.

6. Using the **LIMIT definition** of the derivative function, calculate $f'(x)$ for $f(x) = \dfrac{4}{x}$.

7. Suppose that $f(x) = x^2 e^{-x}$.

   (a) Find $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$.

   (b) Calculate and simplify $f'(x)$ and $f''(x)$. *Hint:* Factor!

   (c) Locate and classify (min, max or neither) the critical points of f.

(d) Locate the inflection points of f.

(e) Using all of the information obtained above, sketch the graph of $f(x)$.

8. You wish to construct a small box by removing four congruent squares from the corners of a 3 inch by 8 inch piece of cardboard. After removing the four corners you fold up the sides to create a box with an open top. What are the dimensions of the box of largest volume you can make in this manner?

9. A manufacturer has been selling 1000 television sets a week at $450 each. A market survey indicates that for each $10 rebate (discount) offered to the buyer, the number of sets sold will increase by 100 per week.

(a) Find the demand function.

(b) How large a rebate should the company offer the buyer in order to maximize its revenue?

(c) If its weekly cost function is $C(x) = 68,000 + 150x$, how should the manufacturer set the size of the rebate in order to maximize its profit?

10. TRUE or FALSE. Decide whether the following statements are true or false. If true, provide an explanation. If false, correct the statement or provide a counterexample. You must provide justification for your answer to receive any credit.

(a) Suppose that $y = f(x)$ is an **exponential** function. If increasing x by one unit increases y by a factor of 3, then increasing x by two units will increase y by a factor of 6.

(b) If a function $f(x)$ is continuous at $x = a$, then it is differentiable at $x = a$.

(c) The graph of $g(x) = f(-x) + 3$ is obtained by shifting the graph of $f(x)$ vertically up by 3 units and reflecting it about the y-axis.

(d) The $\lim\limits_{h \to 0} \dfrac{\sin h - h}{h^2}$ does not exist.

(e) If $s(t) = 2t^3 - 15t^2 + 24t + 16$ represents the position (in feet) of a ball at time t, the total distance traveled by the ball from $t = 0$ to $t = 4$ is 38 feet.