MATH 125, Fall 2007

Applications to Business and Economics (Section 4.7)

Key Terms and Concepts:

1. **Cost Function** $C(x)$. This function represents the cost of producing x units of a product. It is typically an **increasing** function (the more you produce, the more it costs) although its concavity can change.

2. **Marginal Cost Function** $C'(x)$. The function $C'(x)$ approximates the additional cost required to produce the $(x+1)$-th item. For example, $C'(100) = 23\$/\text{item}$ means that it will cost another \$23 to produce the 101st item. Thus, $C'(100)$ is an approximation for $C(101) - C(100)$. The term "marginal" in economics is almost always a synonym for the derivative.

3. **Average Cost Function** $c(x) = \dfrac{C(x)}{x}$. This function describes the cost per unit when x units are produced. You want this function to be as **small** as possible. The formula comes from a typical average calculation where you divide by the total number of items. Note that if the average cost is greater than the marginal cost, you should continue to produce more items because it is cheaper to produce that next item than usual. This will lower your average cost — a good thing! On the other hand, if the average cost is less than the marginal cost, you should stop producing more items because you will be increasing your average cost.

   This leads to our first fundamental rule:

   $$\boxed{\text{The average cost is minimized when marginal cost} = \text{average cost.}}$$

   We will prove this using calculus in class.

4. **Price Function** or **Demand Function** $p(x)$. This function represents the price per unit if you sold x units of a product. For example, $p(200) = \$4$, means that if you put 200 units on the market, you will sell them each for \$4. Note that this is not what you earn in total revenue. If you sold all 200 items then you would have earned $\$4 \cdot 200 = \$800$. This function is usually **decreasing**, since the more you put on the market, the cheaper it will need to be offered in order for people to pay for it. Alternatively, as prices go down, you expect demand to go up, so as x gets bigger, p must be smaller.

5. **Revenue Function** or **Sales Function** $R(x) = xp(x)$. This function represents the total amount of revenue earned if x units are sold at a price of $p(x)$, as indicated in the previous example. You must multiply x times $p(x)$ to obtain the total amount of revenue.

6. **Marginal Revenue Function** $R'(x)$. As with marginal cost, $R'(x)$ represents the additional amount of money earned for selling the $(x+1)$-th item.

7. **Profit Function** $P(x) = R(x) - C(x)$. The function $P(x) = R(x) - C(x)$ represents the total profit earned for selling x items because it is the difference in your revenue and cost. Obviously you would like to **maximize** this function. If P is positive, then you are earning a profit while if P is negative, you are losing money.

8. **Marginal Profit Function** $P'(x)$. This function represents the profit earned for producing and selling the $(x+1)$-th item. If it is positive, then you should continue to produce and sell because you will increase your profits. If it is negative, you should stop because you are not earning as much for producing the next item.

   To maximize profit, we use calculus and quickly see that $P'(x) = 0$ precisely when $R'(x) = C'(x)$. This leads to our second fundamental rule:

   $$\boxed{\text{Profit is at a maximum when marginal revenue = marginal cost.}}$$

   This makes intuitive sense. If your marginal revenue is greater than your marginal cost, you will make more money by selling the next item and should continue to do so. But if your marginal revenue goes below marginal cost, you will only lose money if you keep producing and selling. Thus, the best situation is when these two margins are equivalent.

   Side note: Note that for x_p to be an actual maximum, the second derivative test shows that $R''(x_p) < C''(x_p)$ is necessary in order for $P(x)$ to be concave down at x_p. This means the rate of increase in marginal revenue is less that the rate of increase in marginal cost. This should be checked to ensure you have an actual maximum!