Probability + Statistics

Nov 19 - In Class Probs

1. a) P(type I error) = P(reject null hypothesis | null hypothesis is true)

$= P(\bar{X} > 25 \text{ or } \bar{X} < 15 \mid \mu = 20)$

$= 2P(\bar{X} > 25 \mid \mu = 20)$ b/c X is symmetric about 20

$= 2P\left(\frac{\bar{X} - 20}{\sqrt{25}/\sqrt{9}} > 3\right)$

$= 2P(Z > 3)$

$= 2(0.0013)$

$= 0.0026$

b) P(type II)

$= P(15 < \bar{X} < 25 \mid \mu = 23)$

$= P\left(-3 < \frac{\bar{X} - 20}{5/3} < 3 \mid \mu = 23\right)$

$= P\left(-3 - \frac{20-23}{5/3} < \frac{\bar{X} - 23}{5/3} < 3 - \frac{20-23}{5/3}\right)$

$= P(-4.8 < Z < 1.2)$

$= \Phi(1.2) - \Phi(-4.8)$

$= \Phi(1.2) - (1 - \Phi(4.8))$

$= 0.8849 - 0$

$= 0.8849$

2. $H_0: \sigma = 0.05$

$H_a: \sigma < 0.05$

Using tests of statistical hypotheses can only find conclusive evidence that alterative hypothesis is true, $\therefore$ make Ha what you want to ~~[illegible]~~ prove.

Type I: We conclude that $\sigma < 0.05$ but σ actually is 0.05

Type II: We conclude that $\sigma = 0.05$ but σ is actally < 0.05

3. $H_0: \mu = 48$ $n = 12$ $\alpha = 0.1$

$H_a = \mu \neq 48$ $\bar{x} = 47.1$

(∴ 2 sided test) $s^2 = 4.7$

a) Reject H_0 if

$$-t(n-1)_{\alpha/2} > \frac{\bar{X} - \mu}{s/\sqrt{n}} \quad \text{or} \quad t(n-1)_{\alpha/2} < \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$-1.796 > \frac{47.1 - 48}{\sqrt{4.7}/\sqrt{12}} \quad \text{or} \quad 1.796 < \frac{47.1 - 48}{\sqrt{4.7}/\sqrt{12}}$$

$$-1.796 > -1.438 \quad \text{or} \quad 1.796 < -1.438$$

Neither are true, fail to reject H_0

b) $\left[\bar{X} - t_{\alpha/2}(n-1)\left(\frac{s}{\sqrt{n}}\right), \bar{x} + t_{\alpha/2}(n-1)\left(s/\sqrt{n}\right)\right)$

$\left[47.1 - t_{0.1/2}(12-1)\left(\sqrt{\frac{4.7}{12}}\right), 47.1 + t_{0.1/2}(12-1)\left(\sqrt{\frac{4.7}{12}}\right)\right]$

$[45.976, 48.224]$

If we were to repeat the experiment a large number of times ~~ale~~ and calculate the confidense interval each time, the true mean would be in 90% of the intervals.

4. a) $H_0: \sigma = 0.25$ $\quad S = 0.37$

$H_1: \sigma \neq 0.25$ $\quad n = 51$

$\therefore$ 2-sided test $\quad \alpha = 0.05$

Assuming S is normally distributed

Reject Ho if:

$$\sigma^2 < \frac{(n-1)s^2}{\chi^2_{\alpha/2}(n-1)} \quad \text{or} \quad \sigma^2 > \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}(n-1)}$$

$$(0.25)^2 < \frac{(51-1)(0.37)^2}{71.42} \quad \text{or} \quad (0.25)^2 > \frac{(51-1)(0.37)^2}{32.36}$$

$$0.0625 < 0.0958 \quad \text{or} \quad 0.0625 > 0.2115$$

$\sigma^2 < \frac{(n-1)s^2}{\chi^2_{\alpha/2}(n-1)}$ So reject Ho!

b) Construct a 2-sided 95% CI for $\alpha = 0.05$.
If ~~the~~ $\sigma = 0.25$ is outside the CI, then reject Ho.

5. a) Reject H_0 if

$$t \geq t_\alpha(n-1)$$
$$3.2 \geq t_{0.05}(15-1)$$
$$3.2 \geq 1.761$$

reject null hypothesis

b)

$$t \geq t_\alpha(n-1)$$
$$1.8 \geq t_{0.01}(9-1)$$
$$1.8 \geq 2.896$$

fail to reject null hypothesis

c)

$$t \geq t_\alpha(n-1)$$
$$-0.2 \geq t_\alpha(24-1)$$
$$-0.2 \geq t_\alpha(23)$$

$t_\alpha(23)$ is always positive, so
fail to reject null hypothesis