

Your name(s):

DAY 3: CONDITIONAL PROBABILITY AND INDEPENDENT EVENTS
SEC 1.3-1.4

1 First Exercises

1. The probability that a lab specimen is contaminated is 0.10. Five samples are checked, and the samples are independent.
 - (a) What is the probability that none is contaminated?
 - (b) What is the probability that two are contaminated?
 - (c) What is the probability that at least two are contaminated?
2. Olin alumnus Leighton manufactured 850 yoga chairs one day, but 50 did not meet specifications. Two chairs are tested at random, without replacement, from the batch. Let A be the event that the first chair selected is defective and let B be the event that the second chair selected is defective. Compute $P(B|A)$ using the definition. Now assume the events are independent and recompute that probability. Are A and B independent events?
3. A chemist analyzes seawater samples for two heavy metals: lead and mercury. Past experience indicates that 38% of the samples taken from near the mouth of a river on which numerous industrial plants are located contain toxic levels of lead or mercury, 32% contain toxic levels of lead and 16% contain toxic levels of mercury. What is the probability that a randomly selected sample will contain toxic levels of lead and mercury? Are the presence of lead and mercury mutually exclusive events? Are they independent events?
4. In a game, a participant is given three attempts to hit a ball. On each try, she either scores a hit (H) or miss (M). The game requires that the player must alternate which hand she uses in successive attempts. Write out the outcome space for this experiment. Her chance of scoring a hit with her right hand is 0.7 and with her left hand is 0.4. Assume that the results of successive attempts are independent, and that she wins the game if she scores at least two hits in a row. If she makes her first attempt with her right hand, what is the probability that she wins the game?
5. Suppose that A and B are independent events such that the probability that neither occurs is a and the probability of B is b . Find $P(A)$. Hint: Consider DeMorgan's Laws, which state that for sets X and Y , $(X \cup Y)' = X' \cap Y'$ and $(X \cap Y)' = X' \cup Y'$.

2 Second Exercises

1. Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist. Determine the probability that a randomly chosen member of this group visits a physical therapist.

2. An insurance company examines its pool of auto insurance customers and gathers the following information: (i) All customers insure at least one car. (ii) 70% of the customers insure more than one car. (iii) 20% of the customers insure a sports car. (iv) Of those customers who insure more than one car, 15% insure a sports car. Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.
3. An actuary is studying the prevalence of three health risk factors, denoted by A , B , and C , within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B , is $1/3$. What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A ? (Hint: Draw a Venn diagram first for events A , B , and C .)
4. Prove that if A and B are independent events, then A' and B' are also independent events.