

Day 8 - In-Class

16-Nov-2009

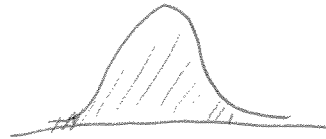
- 1 a. Longer - we
need bigger interval
for more confidence

b. Yes \Rightarrow definition of confidence interval - see p 83

c. No \Rightarrow not the defn of confidence interval

2. 25 samples, 24 degrees of freedom

$$t_{0.05}(24) = +1.711$$



$$\begin{aligned} \mu_{.45} &= 4.05 + 1.711 \cdot \frac{s}{\sqrt{n}} \\ &= 4.05 + 1.711 \cdot \frac{.08}{\sqrt{25}} \\ &= 4.0774 \end{aligned}$$

confidence bound:

$$[-\infty, 4.0774]$$

3 Because we know $\sigma = 2 \text{ MPA}$, we know the
population standard deviation so we should use Z distribution

In the twenty samples,

$$\bar{x} = 9.723$$

$$\cancel{Z_{0.975} = 1.96} \quad Z_{0.975} = 1.96$$

$$\mu_{.975} = \left[\bar{x} - 1.96 \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \cdot \frac{\sigma}{\sqrt{n}} \right] = [9.723 - 1.96 \cdot \frac{2}{\sqrt{20}}, 9.723 + 1.96 \cdot \frac{2}{\sqrt{20}}]$$

$$= \left[9.723 - 1.96 \cdot \frac{2}{\sqrt{20}}, \quad 9.723 + 1.96 \cdot \frac{2}{\sqrt{20}} \right]$$

$$= [8.85, \quad 10.6]$$

4. Binomial distribution:

$$p = 0.52$$

$$\sigma = \frac{p \cdot (1-p)}{\sqrt{n}} = 0.025$$

$$\hat{p} = p \pm 1.96 \cdot 0.025$$

$$\hat{p} = \left[\begin{array}{c} 0.47 \\ 0.471 \end{array}, \quad \begin{array}{c} 0.57 \\ 0.564 \end{array} \right]$$

Right now, the confidence interval is ± 0.05 we want to reduce it by a factor of 2 so we must increase sample size by 4. Thus, we need 400 samples

$$\sigma = \frac{.52 \cdot .48}{\sqrt{400}} = 0.0125$$

$$\hat{p} = p \pm 2 \cdot 0.0125$$

$$= [0.495, \quad 0.545]$$

5. $n = 5$
4 degrees of freedom

$$\sigma^2 = 9$$

confidence interval for σ^2

$$\hat{\sigma}^2 = \left[\frac{\sigma^2}{\chi^2_{.05}(n-1)}, \frac{\sigma^2}{\chi^2_{.95}(n-1)} \right]$$

$$= \left[\frac{1}{0.711}, \frac{1}{9.488} \right]$$

$$\hat{\sigma}^2 = [.1054, 1.41]$$

The sample mean is 3 and sample variance is 0.81. The sample variance is within the confidence interval so there isn't sufficient evidence to disprove the manufacturer's claim. Sorry about all the edits. Everything is in terms of variance, not standard deviation.

$$6. \quad \bar{X}_1 - \bar{X}_2 = 64 - 69 = -5$$

$$\hat{S} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = \sqrt{\frac{52}{11} + \frac{71}{14}} = 3.13$$

$$99\% \text{ confidence} = \bar{X}_1 - \bar{X}_2 \pm t_{23} \cdot \hat{S}$$

$$\text{for } \bar{X}_1 - \bar{X}_2 = -5 \pm 2.807 \cdot 3.13$$

$$= [-13.8, 3.79]$$

Insufficient evidence because 99% confidence interval does not

