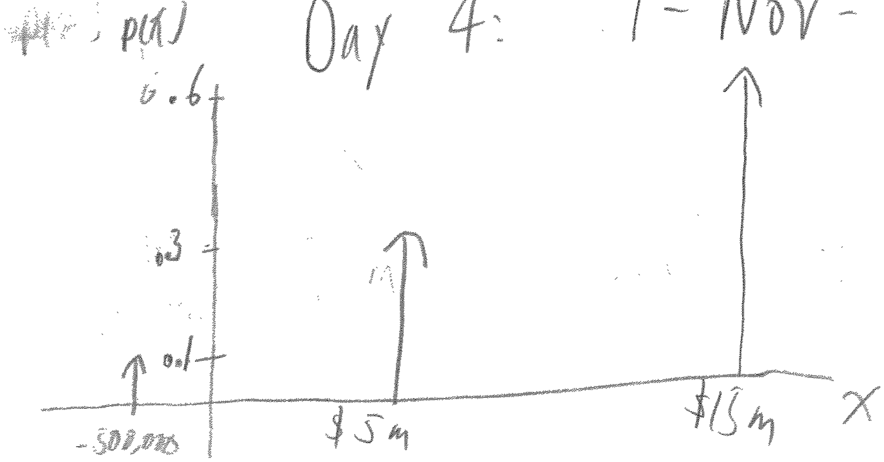
# In-Class Problems:

## Day 4: 1-Nov-2009

1.




2. $E(X_1) = \sum x \cdot p(x)$

For Team1: $= -500{,}000 \cdot .1 + 5 \cdot 10^6 \cdot .3 + 15 \cdot 10^6 \cdot .6$

$= \$10{,}450{,}000$

$E(X_2) = \$7{,}000{,}000$ $\qquad \mu(x) = E(x)$

For team 2

$Var(X_1) = \sum (X - E(X))^2 \cdot p(x)$

$= 0.1 \cdot (-500{,}000 - 10{,}450{,}000)^2 + 0.3 \cdot (5 \cdot 10^6 - 10.45 \cdot 10^6)^2 + 0.6\,(15 \cdot 10^6 - 10.45 \cdot 10^6)^2$

$= 2.13 \cdot 10^{13}$

I would pick Team 2 since they have less risk

$Var(X_2) = 0$

units are in dollar squared.
Standard deviation is a better interpretation

$\sigma(X_1) = \sqrt{Var(X_1)} = 4.62 \cdot 10^6$ $\qquad \sigma(X_2) = 0$

3. $p_k$ is a PMF if

$$\sum_{k=1}^{\infty} (1-p)^k \cdot p = 1$$ for all $p$

$$\sum_{k=1}^{\infty} (1-p)^k \cdot p = p \cdot \sum_{k=1}^{\infty} (1-p)^k$$

$$= p \cdot \frac{1}{1-(1-p)}$$ by convergence of a geometric series

$$= p \cdot \frac{1}{p}$$

$$= 1$$

4. $$\text{Moment} = \sum_{k=1}^{k=n} m_k \cdot r_k^2$$

Moment = ~~mass~~ sum of mass times the distance squared

$$= \boxed{\sum_{k=1}^{n} \frac{1}{2^k} \cdot k^2}$$

$m_k$ is mass of block
$r$ is distance from the ~~moment~~ pivot of rotation

$$\text{Center of Mass} = \frac{1}{M} \sum_{k=1}^{n} m_k \cdot r$$

$M$ is sum of masses

$$= \frac{\sum_{k=1}^{n} \frac{1}{2^k} \cdot k}{\sum_{k=1}^{n} \frac{1}{2^k}}$$

5. $E(\$) = 15 \cdot \frac{2}{13} + 5 \cdot \frac{2}{13} - x \cdot \frac{9}{13}$

payout of J or Q — probability of jack or queen — payout of K or A — prob of K or A

$$\boxed{E(\$) = \frac{40}{13} - \frac{9x}{13}}$$

$E(\$) > 0$

$\frac{40}{13} - \frac{9x}{13} > 0$

$40 > 9x$

$$\boxed{\frac{40}{4} > x}$$

6. $p(X=5) = \frac{8}{20}$ → # of opportunities to draw 5 first + # of opportunities to draw 5 second

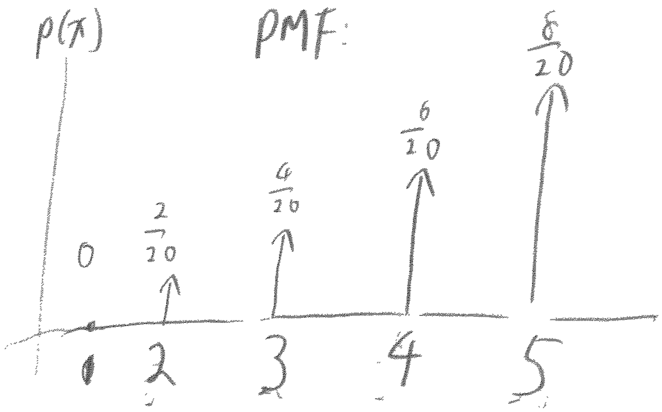
possible options

$p(X=4) = \frac{6}{20}$ → opportunities to draw 4 and not 5

$p(X=3) = \frac{4}{20}$

$p(X=2) = \frac{2}{20}$

$p(X=1) = 0$



$$\mu = \sum x \cdot p(x)$$

$$= 1 \cdot 0 + 2 \cdot \frac{2}{20} + 3 \cdot \frac{4}{20} + 4 \cdot \frac{6}{20} + 5 \cdot \frac{8}{20}$$

$$= \boxed{4}$$

$$\sigma^2 = \sum p(x)(x-\mu)^2$$

$$= \frac{2}{20} \cdot (2-4)^2 + \frac{4}{20} \cdot (3-4)^2 + \frac{6}{20}(0)^2 + \frac{8}{20} \cdot (5-4)^2$$

$$= \boxed{1}$$

7. $P(X=0) = 1 - P(X \geq 1) = 1 - \frac{5}{9} = \frac{4}{9}$

$$P(X=0) = \binom{2}{0} \cdot p^0 \cdot \underset{\substack{\uparrow \\ \text{2 trials}}}{(1-p)^2} = (1-p)^2$$

$$P(X=0) = \frac{4}{9} = (1-p)^2$$

$$p = 1/3$$

$P(Y \geq 1) = 1 - P(Y=0) = 1 - \binom{4}{0} p^0 \cdot (1-p)^4 \leftarrow$ 4 trials

$$= 1 - \left(\frac{2}{3}\right)^4$$

$$= \boxed{\frac{65}{81}}$$