

Your name(s):

DAY 6: ESTIMATION AND LINEAR FUNCTIONS OF INDEPENDENT RVs  
SEC 2.4-2.5

## 1 First Exercises

1. Let  $X_1, X_2, \dots, X_k$  be a random sample of size  $k$  where each  $X_i$  follows a binomial distribution with parameters  $n$  and  $p$ . Use the Method of Maximum Likelihood (ML) to find the ML estimator for  $p$ .
2. Let  $X_1, X_2, \dots, X_k$  be a random sample of size  $k$  where each  $X_i$  follows a Poisson distribution with parameter  $\lambda$ . Use the Method of Maximum Likelihood (ML) to find the ML estimator for  $\lambda$ .
3. Consider two individuals who each toss a coin until the first heads appears. Let  $X_1$  and  $X_2$  denote the number of time that persons  $A$  and  $B$  toss their coins until a heads appears, respectively. A heads occurs with probability  $p$  while a tails occurs with probability  $q = (1-p)$ .
  - (a) What type of probability distribution do  $X_1$  and  $X_2$  both follow? Write the PMF for each.
  - (b) Explain why  $X_1$  and  $X_2$  should be independent RVs. Write their joint PMF  $f(x_1, x_2)$ .
  - (c) Now consider the RV  $Y = X_1 - X_2$ , the difference in the number of tosses. On average, how many tosses should person  $A$  and  $B$  differ by?
  - (d) Compute the variance for  $Y, V(Y)$ . Does the random variable  $Y$  have more or less variability than either  $X_1$  or  $X_2$ ? Can you hypothesize why?
4. Let  $p$  equal the proportion of Americans who favor the death penalty. If in a random sample of 1234 individuals, 864 Americans said they favor the death penalty, find an approximate 95% confidence interval for  $p$ .

## 2 Second Exercises

5. Suppose that it takes an individual playing basketball five attempts to sink their first free throw. Use the maximum likelihood method to estimate  $p$ , the probability of this individual making a single free throw on any one attempt.
6. An oil company wishes to estimate how likely it is to strike oil when a single well is drilled. By experimentation, the company strikes oil for the 3rd time on the 7th well it drills. Use the maximum likelihood method to estimate  $p$ , the probability of successfully striking oil when a single well is drilled.
7. A manufacturer of soap powder packages the soap in “6-pound” boxes. To check the filling machine, they took a sample of  $n = 25$  boxes and weighed them. Given that  $\bar{x} = 5.82$  pounds and  $s = 0.2$  pounds, what can you conclude about the reliability of the machine used to fill the soap boxes?