Block Print Your Name:

Probability and Statistics, Fall 2009, Quiz 4

Sections 3.5-3.7, 4.1-4.2, Estimation, CLT, Statistical Inference, Confidence Intervals

- No resources are allowed, except for a copy of the statistical tables and a calculator/computer for basic arithmetic; do not use any pre-programmed formulas.
- Explain your answers in order on additional sheets of paper as needed.
- There is a strict 90 minute limit for this quiz. Set an alarm. (A goal should be to finish in 60 minutes.)

  Initial Quiz Download/View (Time and Date):
  End of Quiz (Time and Date):

The following formulas might be helpful. If you use them, explain your choices.

$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

$T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$, $(n-1)$ degrees of freedom

$\chi^2 = (n-1)S^2/\sigma^2$, $(n-1)$ degrees of freedom

$Z = \frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\sigma_X^2/n+\sigma_Y^2/m}}$

$T = \frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\frac{(n-1)S_X^2+(m-1)S_Y^2}{n+m-2}(1/n+1/m)}}$, $(n+m-2)$ degrees of freedom

$F = \frac{S_X^2/\sigma_Y^2}{S_Y^2/\sigma_X^2}$, $(n-1)$ numerator degrees of freedom and $(m-1)$ denominator degrees of freedom

1. Consider a random variable X whose probability density function is given by $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$. Determine the Maximum Likelihood Estimator for the parameter λ, explaining all of your work.

2. Suppose you have a random variable X with a continuous uniform distribution given by $f(x) = 1/2$ if $4 \leq x \leq 6$ and $f(x) = 0$ otherwise. Suppose you take random samples of size 100 and compute sample means for each random sample of size 100. We are interested in these sample means. Sketch the graph of the probability density function for the sample means ($\bar{X}$) and include specific comments about the mean and variance of $\bar{X}$.

3. Suppose that you ask 49 Olinites what their favorite real number is. From your collected data, you notice that the sample mean is 127 and the sample standard deviation is 52. You suspect that it will be hard to deduce μ, the true mean of the continuous random variable representing the favorite numbers of every person at Olin, but you'd like to create a 60% confidence interval for μ using your sample of size 49.

   (a) Find a 60% CI for μ, describing your work in detail.

   (b) If you sampled 49 Olinites 100 times, and each time found a 60% CI using your sample mean, how many of the CIs would you expect to capture the real μ? (Ignore the fact that Olin is a small place.)

4. A random sample of 985 likely voters was taken. 592 indicated that they approved of a reduction in the federal income tax. Construct a 95% confidence interval for the true proportion p of voters who approve of reducing the income tax.

5. This problem reproduces the table of heights for male and female Prob/Stat students collected in the spring of 2007.

GROUP	n	$\bar{x}$	s
Males	16	71.1	2.43
Females	13	64.8	2.39

We want to compare the variation in height between males and females.

(a) Using the data above, construct a 95% confidence interval for σ_M, the population standard deviation in height for males. Explain your work, and specifically comment on what statistic is used to construct the interval and how many degrees of freedom are in its distribution.

(b) Repeat part (a) for σ_F, the population standard deviation in height for females.

(c) Now compute a 95 % CI for the ratio of variances (male in the numerator, females in the denominator). Explain your work, including a specific comment about what statistic you are using.

(d) Is there any evidence to support the contention that there is a significant difference in variability in height between males and females? Please comment on how you can use information from (a), (b), and (c) to evaluate the contention.