# Probability + Statistics
## In-Class Problems
## Nov. 5

1. $$\Pr(X \leq K) = \sum_{n=1}^{K} (1-p)^{n-1} p$$

$$= p \cdot \frac{1-(1-p)^K}{1-(1-p)}$$

$$= 1-(1-p)^K$$

2. Binomial $n = 15$

$X$ = # of correctly answered questions

$P(\text{success}) = p = 1/5$

$P(\text{failure}) = 1-p = 4/5$

$$P(X \geq 10) = 1 - P(X < 10)$$

$$= 1 - P(X \leq 9)$$

$$= 1 - \sum_{x=0}^{9} \binom{15}{x} (0.2)^x (0.8)^{15-x}$$

$$\approx 1 - (0.9999) \longleftarrow \text{look in tables}$$

$$\approx 0.0001$$

3. a) Binomial $\quad P(X=K) = \binom{n}{K}(0.5)^K(0.5)^{n-K}$

$P(\text{girl}) = 0.5$

$X$ = number of daughters

b) $P(2 \text{ girls}, 2 \text{ boys}) = P(2\text{girls} \cap 4 \text{ children})$

$= P(2\text{girls} \mid 4 \text{ children}) \cdot P(4 \text{ children})$

$$= \underbrace{\binom{4}{2}(0.5)^2(0.5)^2}_{\text{from part a)}} \cdot \underbrace{\frac{(2.25)^4 e^{-2.25}}{4!}}_{\text{Poisson Dist}}$$

c) $P(\text{children both sexes}) = 1 - P(\text{only one sex})$

$= 1 - [P(\text{all boys}) + P(\text{all girls})]$

$= 1 - [2P(\text{girls})]$

$$= 1 - 2\left[\sum_{n=0}^{\infty} \frac{2.25^n e^{-2.25}}{n!} \cdot \binom{n}{n}(0.5)^n(0.5)^0\right]$$

$$= 1 - 2\left[\sum_{n=0}^{\infty} \frac{2.25^n \cdot e^{-2.25} \cdot (0.5)^n}{n!}\right]$$

4\. Binomial

$P(X=6) = \binom{20}{6} p^6 (1-p)^{14}$

$n = 20$

$X$ = # of individuals who favor the policy

$P(\text{favor policy}) = p$

$\ln P(X=6) = \ln\binom{20}{6} + 6\ln p + 14\ln(1-p)$

$\frac{d}{dp}$

$\frac{d}{dp}(\ln P(X=6)) = \frac{6}{p} - \frac{14}{1-p}$

at maximum, $\frac{d}{dp} P(X=6) = 0$

$0 = \frac{6}{p} - \frac{14}{1-p}$

$\frac{14}{1-p} = \frac{6}{p}$

$p = 6/20$ ← critical point

Check to see if it's a max

$\frac{d^2}{dp^2}\ln(P(X=6)) = -\frac{6}{p^2} - \frac{14}{(1-p)^2}$

which is always negative

$\therefore \hat{p} = 6/20$

5\. $X_1$ = front tire replacements $(\lambda_1 = 2 \frac{\text{times}}{\text{year}})$

$X_2$ = back tire replacements $(\lambda_2 = 1.5 \frac{\text{times}}{\text{year}})$

$P(2 \text{ tires replaced}) = P(X_1=2)P(X_2=0) + P(X_1=1)P(X_2=1) + P(X_1=0)P(X_2=2)$

$$= \frac{e^{-\lambda_1}\lambda_1^2}{2!} \cdot \frac{e^{-\lambda_2}\lambda_2^0}{0!} + \frac{e^{-\lambda_1}\lambda_1^1}{1!} \cdot \frac{e^{-\lambda_2}\lambda_2^2}{1!}$$

$$+ \frac{e^{-\lambda_1}\lambda_1^0}{0!} \cdot \frac{e^{-\lambda_2}\lambda_2^2}{2!}$$

6. $X$ = trial on which the $r^{th}$ success occurs

$$P(X=x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

$r=3$

$x=7$

$$P(X=7) = \binom{6}{2} p^3 (1-p)^4$$

$$\hookrightarrow \ln(P(X=7)) = \ln\binom{6}{2} + 3\ln p + 4\ln(1-p)$$

$$\frac{\frac{d}{dp}(P(X=7))}{P(X=7)} = \frac{3}{p} - \frac{4}{1-p}$$

At max, $d(P(X=7)) = 0$

$$0 = \frac{3}{p} - \frac{4}{1-p}$$

$p = 3/7 \longleftarrow$ critical point!

check 2nd derivative to see if it's a max or min

$$\frac{d^2}{dp^2} \ln(P(X=7)) = -\frac{3}{p^2} - \frac{4}{(1-p)^2}$$

which is always negative

$\therefore p = 3/7$ is a max!

7. $\hat{p} = \frac{864}{1234} = 0.70$

$$CI = 0.70 \pm 2\sqrt{(0.70)(0.30)/1234}$$

or $[0.674, 0.726]$