Solutions to Post-Class Problems Due 2/5

2.2

① $E(x) = \frac{1}{5}(1) + \frac{1}{5}(2) + \frac{1}{5}(3) + \frac{1}{5}(4) + \frac{1}{5}(5) = 3$

$E(x^2) = \frac{1}{5}(1)^2 + \frac{1}{5}(2)^2 + \frac{1}{5}(3)^2 + \frac{1}{5}(4)^2 + \frac{1}{5}(5)^2 = 11$

$E[(x+2)^2] = E(x^2+4x+4) = E(x^2) + \underbrace{E(4x)}_{4E(x)} + E(4) =$

$11 + 4(3) + 4 = 27$

③ $E(x) = -1\left(\frac{(|-1|+1)^2}{9}\right) + 0\left(\frac{0^2}{9}\right) + 1\left(\frac{2^2}{9}\right) = 0$

$E(x^2) = (-1)^2\left(\frac{2^2}{9}\right) + 0^2\left(\frac{0^2}{9}\right) + (1)^2\left(\frac{2^2}{9}\right) = 8/9$

$E(3x^2-2x+4) = 3E(x^2) - 2E(x) + E(4) = 3(8/9) - 2(0) + 4$

$= \frac{24}{9} + \frac{36}{9} = \frac{8}{3} + \frac{12}{3} = \frac{20}{3}$

⑦ a) Average Class Size $= \frac{\text{total students}}{\text{total classes}} = \frac{1000}{20} = 50$

b) $X = 25, 100, 300$

$$f(x) = \begin{cases} 0.4, & x = 25 \\ 0.3, & x = 100 \\ 0.3 & x = 300 \end{cases} \longleftarrow \frac{\#\text{ of students in 25 person classes}}{1000 \text{ total students}} = \frac{16 * 25}{1000} = \frac{400}{1000} = 0.4$$

c) $E(x) = 0.4(25) + 0.3(100) + 0.3(300) = 130$

Yes. This is surprising because it's not the same as the average class size.

part (a) looks for average class size, but part (b/c) look for the probability that a student is in a class of a certain size.

2.3

9 a) $P(X<5) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$$= \binom{60}{0}.95^{60} + \binom{60}{1}.95^{59}(.05) + \binom{60}{2}.95^{58}(.05)^2 + \binom{60}{3}.95^{57}(.05)^3$$

$$+ \binom{60}{4}.95^{56}(.05)^4 = 0.8197$$

b) $\lambda = n \cdot p = 60 \cdot 0.05 = 3$

$P(X \leq 4) = 0.8153$ use Tables in Book

same as $P(X<5)$ since discrete

11 $\lambda = 11$

$P(X>10) = 1 - P(X \leq 10) = 1 - 0.460 = 0.540$

17 a) $\mu = \frac{n}{p} = \frac{10}{0.6} = 16.67$

$\sigma^2 = \frac{n(1-p)}{p^2} = \frac{10(0.4)}{0.6^2} = 11.1$

$\sigma = \sqrt{11.1} = 3.33$

negative binomial

b) $P(X=16)$

$\binom{15}{9}(.6)^9(.4)^6 * (0.6) = 0.1240$

9 successes in first 15 trials

P(success) on 16th trial

19 Use hypergeometric distribution

one class = good other class = defective

$$f(x) = \frac{\binom{20}{x}\binom{180}{10-x}}{\binom{200}{10}} ; \; x = 0, 1, \ldots, 10$$ ← # of defective fuses out of 10 tested

$n = 10$ $N_2 = 180$
$N_1 = 20$ $N = 200$
$p = 1/10$

$$\mu = n\left(\frac{N_1}{N}\right) = np = 10\left(\frac{20}{200}\right) = 1$$

$$\sigma^2 = n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right) = np(1-p)\left(\frac{N-n}{N-1}\right) = 10\left(\frac{1}{10}\right)(0.9)\left(\frac{190}{199}\right) = 0.8593$$

hypergeometric: $P(X=0)$

$$f(0) = \frac{\binom{20}{0}\binom{180}{10}}{\binom{200}{10}} = 0.3398$$

binomial: $p = 0.10$

$$f(0) = \binom{10}{0}(0.10)^0(0.90)^{10} = 0.3487$$

poisson: $\lambda = \mu = 1$

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad f(0) = \frac{1^0 e^{-1}}{0!} = 0.368$$