Your name(s):

Day 7: Continuous Probability Distributions
Sec 3.2-3.3

1. If a parachutist lands at a random point on a line between markers A and B, find the probability that he is closer to A than to B. Find the probability that his distance to A is more than three times his distance to B. Now assuming that three parachutists operate independently, what is the probability that exactly one of the three lands past the midpoint between A and B?

2. The time until failure X of an incandescent light bulb follows an exponential probability density with expected value 1000 hours.

   - Derive an expression for the CDF $F(x)$ of the RV X using the definition $F(x) = \int_{-\infty}^{x} f(t)\, dt$.
   - Calculate the probability that a light bulb will last no more than 2000 hours, given that it has already lasted 1000 hours.

3. A continuous random variable X follows a Weibull distribution if it's PDF is given by

$$f(x) = \begin{cases} \frac{1}{\alpha} m x^{m-1} e^{-x^m/\alpha} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

   - Show that $\int_{-\infty}^{\infty} f(x)\, dx = 1$
   - Find an equation for the median of the Weibull distribution as a function of m and α.

4. The percentage of impurities per batch in a chemical product is a random variable X with PDF

$$f(y) = \begin{cases} 12x^2(1-x) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   A batch with more than 40% impurities cannot be sold. What is the mean percentage of impurities per batch? Given that a randomly selected batch does not contain more than the mean percentage of impurities, what is the probability that it cannot be sold?

5. Find the three quartiles for the following probability distribution:

$$f(x) = \frac{1}{\pi(1+x^2)} \quad -\infty < x < \infty$$

6. Let X be a χ^2 random variable with 10 degrees of freedom.

   - Find a and b such that $P(a < X < b) = 0.95$.
   - Find a such that $P(X < a) = 0.1$.
   - Find b such that $P(X > b) = 0.01$.