

$$\begin{aligned}
 1) \quad a) \quad & P(16-20) = .08 & P(A|16-20) &= .06 \\
 & P(21-30) = .15 & P(A|21-30) &= .03 \\
 & P(31-65) = .49 & P(A|31-65) &= .02 \\
 & P(\geq 66) = .28 & P(A|\geq 66) &= .04
 \end{aligned}$$

$$\begin{aligned}
 P(A) &= P(A|16-20)P(16-20) + P(A|21-30)P(21-30) + P(A|31-65)P(31-65) \\
 &\quad + P(A|\geq 66)P(\geq 66) \\
 &= (.06)(.08) + (.03)(.15) + (.02)(.49) + (.04)(.28) = \boxed{.0303}
 \end{aligned}$$

$$b) \quad P(16-20|A) = \frac{P(A|16-20)P(16-20)}{P(A)} = \frac{(.06)(.08)}{.0303} = \boxed{.1584}$$

$$2) \quad a) \quad \text{NEG. BINOMIAL} \quad f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r} \quad \begin{array}{l} X \equiv \text{NO. OF TRIALS ON WHICH} \\ \text{RTH SUCCESS OCCURS} \end{array}$$

SINCE RTH SUCCESS IS ON LAST TRIAL (XTH), MUST HAVE R-1 SUCCESSSES IN PREVIOUS X-1 TRIALS, SO $X = r, r+1, \dots$

$p \equiv \text{PROB. OF SUCCESS}$
 $(1-p) \equiv \text{PROB. FAILURE}$

$$b) \quad P(X=5) = f(5) = \binom{4}{2} (.7)^3 (.3)^2 = 6(.7)^3 (.3)^2 = \boxed{.18522}$$

$X=5; r=3$

c) $A \equiv \text{MAKE 3RD SHOT ON 5TH ATTEMPT}; B \equiv \text{MAKE 5TH SHOT ON 7TH ATTEMPT.}$

$$\frac{P(B|A) = P(B \cap A)}{P(A)} = \frac{(.18522)(.7)(.7)}{(.18522)} = (.7)^2 = \boxed{.49}$$

$$3) \quad a) \quad P(X \leq 95) = P\left(\frac{X - \mu}{\sigma} \leq \frac{95 - 115}{12}\right) = P(Z \leq -1.67) = \boxed{.0475}$$

b) USE BINOMIAL, $n=300, p=.0475, (1-p)=.9525$

$$P(X \leq 3) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \binom{300}{0} (.0475)^0 (.9525)^{300} + \binom{300}{1} (.0475)^1 (.9525)^{299} + \binom{300}{2} (.0475)^2 (.9525)^{298}$$

$$= (.9525)^{288} \left((.9525)^2 + 300(.0475)(.9525) + 44850(.0475)^2 \right)$$

$$= (.9525)^{288} (115.673) = \boxed{.00000947}$$

$$4) a) \mu = E(Y) = \int_1^{\infty} y \frac{3}{y^4} dy = \int_1^{\infty} 3y^{-3} dy = 3 \frac{y^{-2}}{-2} \Big|_1^{\infty} = -\frac{3}{2} \left(\frac{1}{y^2} \Big|_1^{\infty} \right) \\ = -\frac{3}{2}(0-1) = \left(\frac{3}{2} \right)$$

$$E(Y^2) = \int_1^{\infty} y^2 \frac{3}{y^4} dy = \int_1^{\infty} 3y^{-2} dy = 3 \frac{y^{-1}}{-1} \Big|_1^{\infty} = -3 \left(\frac{1}{y} \Big|_1^{\infty} \right) = -3(0-1) = 3$$

$$\text{SO } \sigma^2 = E(Y^2) - \mu^2 = 3 - \frac{9}{4} = \frac{12}{4} - \frac{9}{4} = \left(\frac{3}{4} \right)$$

b) $n=5$ $\bar{Y} \equiv \text{AUG COMPRED}$

$$P(\bar{Y} \geq 3.5) = P\left(\frac{\bar{Y} - \mu}{\sigma/\sqrt{n}} \geq \frac{3.5 - 1.5}{.75/\sqrt{5}}\right) = P(Z \geq \frac{2/\sqrt{5}}{.75}) = P(Z \geq 6.96) \\ = 0$$

$$5) n=10 \quad \alpha=.1 \quad \bar{X} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \Rightarrow 39.1 \pm t_{.05} \frac{(17.3)}{\sqrt{10}} \\ \bar{X}=39.1 \quad S=17.3 \quad \underline{90\%} \Rightarrow 39.1 \pm \underbrace{(1.833)}_{(10.027)} \frac{(17.3)}{\sqrt{10}} \begin{cases} 49.1278 \\ 29.0721 \end{cases}$$

SO, THE 90% CI IS $(29.0721, 49.1278)$

b) YES, CLAIM IS VALID SINCE $\mu=30 \in (29.0721, 49.1278)$

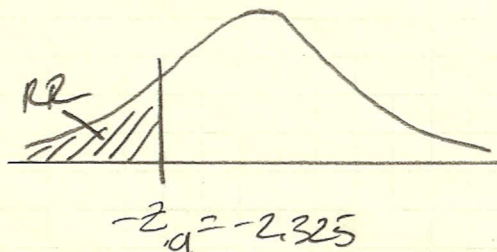
$$6) n=100 \quad \mu=120 \\ \bar{X}=115 \quad \alpha=.01 \\ S=24$$

$$H_0: \mu=120 \\ H_a: \mu < 120$$

$$\text{USE } Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \approx \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

SINCE n IS LARGE!

$$Z = \frac{115-120}{24/\sqrt{100}} = \frac{(-5)(10)}{24} = -2.083$$



SINCE $Z = -2.083 > -z_{\alpha}$, IT IS NOT IN REJ. REGION

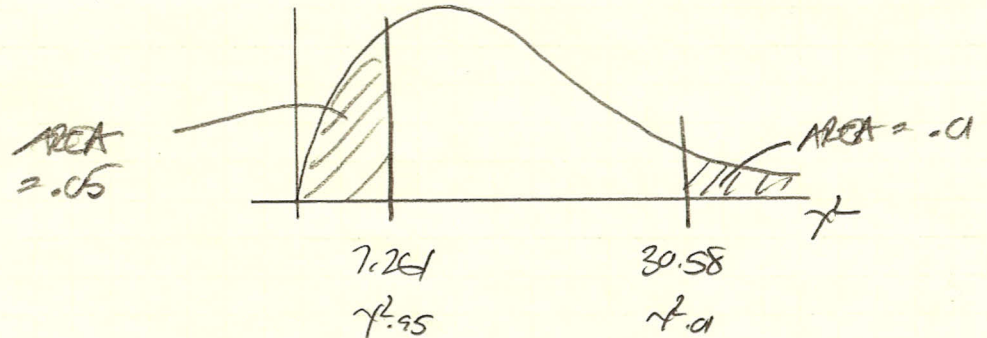
\therefore FAIL TO REJECT H_0 . NO DIFF. IN B.W. BETW. SES BABIES & POP. IN GENERAL

7) $\sigma^2 = 25$
 $n = 16$
 (15 Dof)

REJECT IF $S^2 > 50.967$ or $S^2 < 12.102$

$$\hookrightarrow \frac{(n-1)S^2}{\sigma^2} > \frac{(15)(50.967)}{25} \text{ or } \frac{(n-1)S^2}{\sigma^2} < \frac{(15)(12.102)}{25}$$

$$\chi^2 > 30.58 \text{ or } \chi^2 < 7.261$$



SO PROB. REJECT = $.05 + .01 = .06$