Probability + Statistics

Oct 29 – In Class Problems

1. a) $\binom{5}{0}(0.9)^5 = 0.59049$

   b) $\binom{5}{2}(0.9)^3(0.1)^2 = 0.0729$

   c) $\binom{5}{2}(0.9)^3(0.1)^2 + \binom{5}{3}(0.9)^2(0.1)^3 + \binom{5}{4}(0.9)^1(0.1)^4$
   $+ \binom{5}{5}(0.1)^5 = 1 - \left[\binom{5}{1}(0.9)^4(0.1) + \binom{5}{0}(0.9)^5\right]$
   $= 0.08146$

2. $A \equiv$ "1^{st} Chair Defective"

   $B \equiv$ "2^{nd} Chair Defective"

   If A and B are independent, then $P(A \cap B) = P(A)P(B)$ and $P(B|A) = P(B)$

$$P(A \cap B) = \underset{1^{st}}{\frac{50}{850}} \cdot \underset{2^{nd}}{\frac{49}{849}}$$

$$P(B|A) = \frac{49}{849}$$

$$P(B) = \begin{cases} \frac{49}{849} & \text{If } A \text{ is defective} \\ \frac{50}{849} & \text{If } A \text{ is not defective} \end{cases}$$

$$P(A) = \frac{50}{850}$$

So $P(B|A) \neq P(B)$

If the first choice is defective, the likelihood of B decreases. Intuitively, A influences B so they can't be independent.

3. P(Lead or Mercury) = P(Lead) + P(Mercury) - P(Lead and Mercury)

$0.38 = 0.32 + 0.16 - P(\text{Mercury and Lead})$

$P(\text{Mercury and Lead}) = 0.1$

4. 3 Ways to win: {HHM, HHH, MHH}

Sum their Probabilities:

$= P(HHM) + P(HHH) + P(MHH)$

$= (0.7)(0.4)(0.3) + (0.7)(0.4)(0.7) + (0.3)(0.4)(0.7$

$= 0.364$

5. Given: $P(A' \cap B') = a$

$P(B) = b$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$1 - P((A \cup B)') = P(A) + P(B) - P(A)P(B)$

$1 - P((A \cup B)') = P(A)(1 - P(B)) + P(B)$

$1 - P(A' \cap B') = P(A)(1 - P(B)) + P(B)$

$1 - a = P(A)(1 - b) + b$

$P(A) = \dfrac{1 - a - b}{1 - b}$

6. $P(\text{Janet} \mid \text{Incomplete}) = \dfrac{P(\text{Incomplete} \mid \text{Janet})\, P(\text{Janet})}{P(\text{Incomplete})}$

$= \dfrac{(0.05)(0.2)}{(0.05)(0.2) + (0.1)(0.6) + (0.1)(0.15) + (0.05)(0}$

$= 0.114$