Preclass 5 - Nov - 2009

Remember, Remember, the Fifth of November

2. Binomial Distribution: $p = 0.7$, $n = 25$

$b(25, 0.7)$

a. $P(X \geq 13)$ for $p = 0.7$ is equivalent to

$P(X \leq 12)$ for $p = 1 - 0.7 = 0.3$

By the tables, this is 0.9825

b. $P(X \leq 11) = 1 - .9940 = .0060$

$\uparrow$ $P(X \leq 13)$ for $p = 0.3$ by the tables

c. $P(X = 12) = P(X \leq 12) - P(X \leq 11)$

$= .9940 - .9825 = 0.0115$

d. $\mu = n \cdot p = 25 \cdot 7/10 = 17.5$

$\sigma^2 = n \cdot p \cdot (1-p) = 25 \cdot \frac{7}{10} \cdot \frac{3}{10} = 5.25$

$\sigma = \sqrt{5.25}$

8. a. Probability of 3 wrong * probability of 1 right

$$\boxed{\left(\frac{4}{5}\right)^3 \cdot \frac{1}{5}}$$

b. $f(x) = \left(\frac{4}{5}\right)^{x-1} \left(\frac{1}{5}\right)$ $\quad x = 1, 2, 3, \ldots$

probability of everything before x being wrong

probability of x being right

$\mu = \frac{1}{p} = 5$ $\rightarrow$ Negative Binomial Distribution

$\sigma^2 = \frac{1-p}{p^2} = 20$

12. $\lambda = \frac{225}{150} = 1.5$

$P(X \leq 1) = \frac{1.5^0 e^{-1.5}}{0!} + \frac{1.5^1 e^{-1.5}}{1!} = \boxed{0.5578}$

2.4

4. a. $X=0$: 17 $X=4$: 49 $X=8$: 1

$X=1$: 47 $X=5$: 28

$X=2$: 63 $X=6$: 21

$X=3$: 63 $X=7$: 11

b. $\bar{X} = \frac{303}{100} = 3.03$

$s^2 = 3.193$

c. Poisson Distribution w/ $\lambda = 3.03$ is a good fit since $\bar{X}$ and s^2 are about equal Negative Binomial is also possible since $\bar{X} < s^2$

6. $\bar{X} = \frac{347}{62} = 5.517$

$s^2 = 3.491$

Binomial seems like the best fit since $\bar{X} > s^2$

2.5

2. $Var(Y) = Var(3X_2 - X_1)$

$25 = 9\, Var(X_2) + Var(X_1)$

$25 = 9 \cdot 2 + k$

$\boxed{k = 7}$

2.5. 4. a. $\mu_Y = E(X_1 \cdot X_2) = E(X_1) \cdot E(X_2)$ → due to independence

$= \mu_1 \cdot \mu_2$

b. $Var(Y) = E[(X_1 X_2 - \mu_1 \mu_2)^2]$

$= E(X_1^2 X_2^2) - 2\mu_1\mu_2 E(X_1 X_2) + \mu_1^2 \mu_2^2$

by part a

$= (\sigma_1^2 + \mu_1^2)(\sigma_2^2 + \mu_2^2) - 2\mu_1\mu_2 \, \mu_1 \cdot \mu_2 + \mu_1^2 \cdot \mu_2^2$

$= \boxed{\sigma_1^2 \sigma_2^2 + \sigma_1^2 \cdot \mu_2^2 + \sigma_2^2 \cdot \mu_1^2} + \cancel{\mu_1^2 \cdot \mu_2^2} - \mu_1^2 \mu_2^2$