Solutions to Post-class Problems Due 29 Jan

1.3

(5) a) {RW, RR, WR, WW}

b) $P(RR \mid \text{red eyes}) = \dfrac{P(RR \text{ and red eyes})}{P(\text{red eyes})} = \dfrac{1/4}{3/4} = \dfrac{1}{3}$

(7) Possible Draws $= \{(O_1,O_2), (O_1,B_1), (O_1,B_2), (O_2,O_1), (O_2,B_1), (O_2,B_2), (B_1,B_2), (B_1,O_1), (B_1,O_2), (B_2,B_1), (B_2,O_1), (B_2,O_2)\}$

10 possible draws where at least 1 is orange. $\frac{2}{10} = 0.2$
2 possible draws where both are orange.

(13) a)

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

b) 8 possible combinations that sum to 7 or 11.
{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (5,6), (6,5)} ∴ 8/36

c) 11 possible ways to roll an 8 or 7
5 ways to roll an 8 ∴ 5/11

d) There are 5 ways to roll an 8 ∴ $P(8) = \frac{5}{36}$
$P(8)\,P(8 \mid 7 \text{ or } 8) = \left(\frac{5}{36}\right)\left(\frac{5}{11}\right)$

e) Following same logic as above:

$$\frac{8}{36} + 2\left[\left(\frac{5}{36}\right)\left(\frac{5}{11}\right) + \left(\frac{4}{36}\right)\left(\frac{4}{10}\right) + \left(\frac{3}{36}\right)\left(\frac{3}{9}\right)\right] = 0.49293$$

P(4)P(4|4or7)
P(10)P(10|10or7)

P(5)P(5|5or7)
P(9)P(9|9or7)

P(6)P(6|6or7)
P(8)P(8|8or7)

1.4

(5) a) P(one player is successful):

$P(A_1 \cap A_2' \cap A_3') + P(A_1' \cap A_2 \cap A_3') + P(A_1' \cap A_2' \cap A_3)$

probability A_1 is successful, A_2 and A_3 are not.

Because they are independent:

$= (0.5)(0.3)(0.4) + (0.5)(0.7)(0.4) + (0.5)(0.3)(0.6) = 0.29$

b) P(two players are successful):

$P(A_1 \cap A_2 \cap A_3) + P(A_1 \cap A_2' \cap A_3) + P(A_1' \cap A_2 \cap A_3)$

$= (0.5)(0.7)(0.4) + (0.5)(0.3)(0.6) + (0.5)(0.7)(0.6) = 0.44$

(7) a) $P(A \cap B \cap C) = (0.5)(0.8)(0.9) = 0.36$

b) $P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C)$

$= (0.5)(0.8)(0.1) + (0.5)(0.2)(0.9) + (0.5)(0.8)(0.9) = 0.49$

c) $P(A' \cap B' \cap C') = (0.5)(0.2)(0.1) = 0.01$

(11) Event A: choose 3 white balls in 1st 5.

Event B: 6th ball is white.

a) $P(A) = {}_5C_3 \cdot 0.5^5 = 0.3125$

$P(B) = 0.5$

$P(A \cap B) = 10 \cdot 0.5^6 = 0.15625$

A and B are independent

b) $P(A) = {}_5C_3 \cdot \dfrac{\overbrace{10 \cdot 9 \cdot 8}^{\text{white balls}} \cdot \overbrace{10 \cdot 9}^{\text{red balls}}}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}$

\# of ways this can happen

\# of balls that can be chosen w/o replacement

$P(B) = \dfrac{7}{15}$ ← 7 white, ← 15 total

c) Neither model is very good.

$P(A \cap B) = 10 \cdot \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 10 \cdot 9}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15}$