Your name(s):

Day 10: Central Limit Theorem, Approximations for Discrete Distributions Sec 3.6-3.7

1. At a manufacturing plant for large diesel engines, cylinders are bored into the engine blocks and pistons are made. The machinery that makes these parts is not perfect. The inside diameter X_1 of cylinders is a normal random variable with mean 30.25 cm and standard deviation 0.06 cm. The outside diameter X_2 of pistons is normally distributed with mean 30 cm and standard deviation 0.05 cm.

   - What is the probability that a randomly chosen piston will not fit into a randomly selected cylinder? (Hint: First define a new random variable and find its distribution).
   - A given piston performs best if the clearance gap between the piston and the cylinder wall is between 0.1 and 0.35 cm. What is the probability that a randomly chosen piston performs optimally?
   - An engine has six pistons. What is the probability that an engine will be made that has at least four pistons that perform optimally?

2. Suppose $T_1, T_2, \ldots T_n$ are independent times until failure of n electronic components. Each failure time has the identical exponential distribution with expected time until failure of $\mu = 1000$. Let $\bar{T}$ be the average time until failure of n such randomly selected machines. Determine an approximate distribution for $\bar{T}$ and make a sketch of it on the the same graph as a sketch of the distribution of the time until failure of a single machine.

3. Shear strength measurements for spot welds have been found to have standard deviation 10 psi. If 100 test welds are to be measured, what is the approximate probability that the sample mean will be within 1 psi of the true population mean?

4. Achievement test scores of all high school seniors in a state have mean 60 and variance 64. A random sample of $n = 100$ students from one large high school had a sample mean score of 58. Is there evidence to suggest that this high school is inferior with respect to the others in the state?

5. Suppose that X has a binomial distribution with $n = 25$ and $p = .4$. Find the exact probabilities that $X \leq 8$ and $X = 8$ and compare these to the corresponding values found by using the normal approximation.

6. Candidate A believes that she can win a city election if she can earn at least 55% of the vote in precinct 1. She also believes that about 50% of the city's voters favor her. If $n = 100$ voters show up to vote at precinct 1, what is the probability that candidate A will recieve at least 55% of their votes?

7. In the interest of pollution control, an experimenter wants to count the number of bacteria per small volume of water. Let X denote the bacteria count per cubic centimeter of water and assume that X is a RV with mean 100. What probability distribution does X follow? If the allowable pollution in a water supply is a count of 110 per cubic centimeter, approximate the probability that a given sample will pass inspection.

8. A pollster believes that 20% of the voters in a certain area favor a bond issue. If 64 voters are randomly sampled from the large number of voters in the area, approximate the probability that the sampled fraction of voters favoring the bond issue will not differ from the true fraction by more than 0.06.