Probability & Statistics

Preclass Problems due 2/9/10
Sec 3.2 #2,6
Sec 3.3 #2,4,6

3.2 2) X is continuous-type random variable with space $S = \{x: 0<x<10\}$

$P(A_1) = 1/4$ where $A_1 = \{x: 1<x<5\}$

$A_1' = \{x: 5<x<10\}$

$A_2 = \{x: 5<x<10\} \subset A_1'$

is a subset of

$P(A_2') = P(A_1') \leq 1 - P(A_1)$

$\leq 1 - 1/4$

$\leq 3/4$

3.2 6) a) $f(x) = \frac{1}{32}(x^3+8), \; -2<x<2$

$\mu = \int_{-2}^{2} x f(x)dx$

$\mu = \int_{-2}^{2} \frac{x}{32}(x^3+8)dx$

$\mu = \frac{1}{32}\int_{-2}^{2} x^4 + 8x\, dx$

$\mu = \frac{1}{32}\left[\frac{1}{5}x^5 + 4x^2\right]_{-2}^{2}$

$\boxed{\mu = 2/5}$

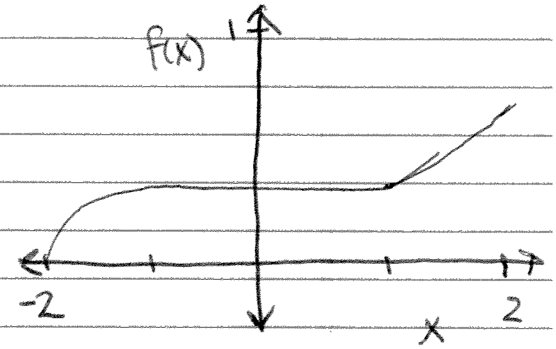
$\sigma^2 = \int_{-2}^{2} x^2 f(x)dx - \mu^2$

$\sigma^2 = \int_{-2}^{2} \frac{x^2}{32}(x^3+8)dx - (2/5)^2$

$\sigma^2 = \frac{1}{32}\int_{-2}^{2} x^5 + 8x^2 dx - 4/25$

$\sigma^2 = \frac{1}{32}\left[\frac{1}{6}x^6 + 8/3x^3\right]_{-2}^{2} - 4/25$

$\boxed{\sigma^2 = 88/25}$



b) $f(x) = \frac{1}{18}(x+2) \quad -2<x<4$

$\mu = \int_{-2}^{4} x f(x)dx$

$\mu = \int_{-2}^{4} \frac{x}{18}(x+2)dx$

$\mu = \frac{1}{18}\int_{-2}^{4} x^2+2x\, dx$

$\mu = \frac{1}{18}\left[\frac{1}{3}x^3 + x^2\right]_{-2}^{4}$

$\boxed{\mu = 2}$

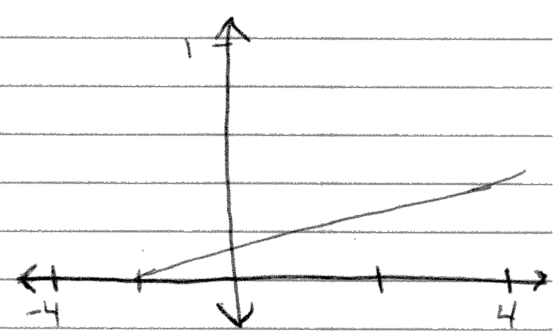
$\sigma^2 = \int_{-2}^{4} x^2 f(x) - \mu^2$

$\sigma^2 = \int_{-2}^{4} \frac{x^2}{18}(x+2)dx - (2)^2$

$\sigma^2 = \frac{1}{18}\int_{-2}^{4} x^3+2x^2 dx - 4$

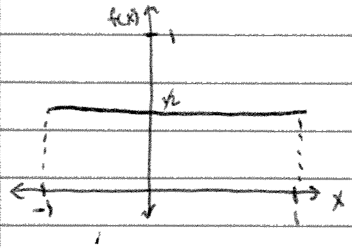
$\sigma^2 = \frac{1}{18}\left[\frac{1}{4}x^4 + 2/3x^3\right]_{-2}^{4} - 4$

$\boxed{\sigma^2 = 2}$

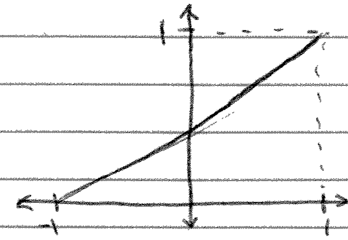


3.3 2) $f(x)=1/2, \quad -1\le x\le 1$

PDF: $f(x)=1/2$



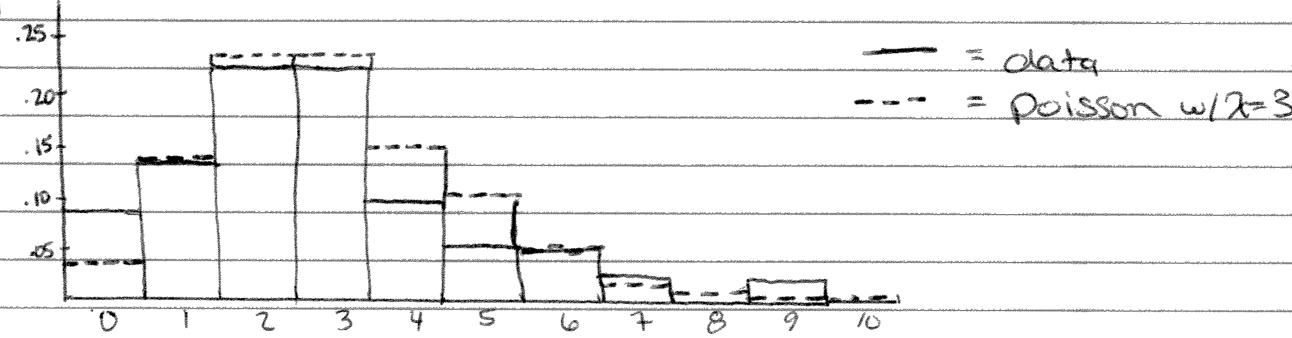
Distribution: $F(x)$



$\mu=\int_{-1}^{1} x f(x)dx$
$\mu=\int_{-1}^{1} \frac{1}{2}x dx$
$\mu=\left[\frac{1}{4}x^2\right]_{-1}^{1} = 0$

$\sigma^2=\int_{-1}^{1} x^2 f(x)dx - \mu^2$
$\sigma^2=\int_{-1}^{1} \frac{1}{2}x^2 dx - 0^2$
$\sigma^2=\left[\frac{1}{6}x^3\right]_{-1}^{1} = 1/3$

3.3 4) a)



b) $\bar{x}=\frac{717}{35}=20.4857$
$S=20.3437$

c) $f(x)=\frac{1}{\theta}e^{-x/\theta}$

$P(X>15)=1-P(X<15)$
$=1-\int_0^{15} f(x)dx$
$=1-\int_0^{15}\frac{1}{\theta}e^{-x/\theta}dx$
$=1-\frac{1}{\theta}\left[-\theta e^{-x/\theta}\right]_0^{15}$
$=1+(e^{-15/\theta}-e^{0})$
$=e^{-15/\theta}$

$\theta=20$
$P(X>15)=e^{-15/20}=0.4724$

Proportion of observations greater than 15:
$49/105 = 0.4667$

3.3 4) Continued

d) $P(X>45.5 \mid X>30.5) = \dfrac{P(X>45.5 \cap X>30.5)}{P(X>30.5)}$

$= \dfrac{P(X>45.5)}{P(X>30.5)}$

$= \dfrac{e^{-45.5/20}}{e^{-30.5/20}}$ See part d if confused where this is from

$= 0.4724$

Compare to: 0.4615

3.3 6) $f(x) = \frac{1}{\theta} e^{-\theta}$

$F(x) = \int_0^x \frac{1}{\theta} e^{-\theta}$ (c.d.f)

$F(x) = 1 - e^{-x/\theta}$

a) $1/4 = F(x) = 1 - e^{-x/\theta}$

$3/4 = e^{-x/\theta}$

$\ln 3/4 = -x/\theta$

$x = -\theta \ln(3/4)$

$x = \theta \ln(4/3)$

b) $\mu = \theta$

$\mu - x_{1/4} = \theta - \theta \ln(4/3)$

$= \theta(1 - \ln(4/3))$

$= \theta(1 + \ln(3/4))$

$\approx 0.7123\theta$

c) $3/4 = F(x) = 1 - e^{-x/\theta}$

$1/4 = e^{-x/\theta}$

$x = \theta \ln(4)$

$x = 1.3863\theta$

d) $q_3 - \theta = \theta[\ln(4) - 1]$

$= 0.3863\theta$