Derivative Rules, Properties and Important Limits

Derivative Rules:

1. Derivative of a Constant
$$\frac{d}{dx}[c] = 0$$
2. Power Rule
$$\frac{d}{dx}[x^n] = nx^{n-1}$$
3. Derivative of the Exponential Function
$$\frac{d}{dx}[e^x] = e^x$$
4. Derivative of the General Exponential Function
$$\frac{d}{dx}[a^x] = a^x \ln a$$
5. Derivative of the Natural Logarithm Function
$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$
6. Derivative of the General Logarithm Function
$$\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$$
7. Derivative of Trigonometric Functions
$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$
$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$
$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$
8. Derivative of Inverse Trigonometric Functions
$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$$
9. Chain Rule (general)
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
10. Chain Rule (power functions)
$$\frac{d}{dx}[u^n] = nu^{n-1} \cdot \frac{du}{dx}$$

11. Chain Rule (exponential function)

$$\frac{d}{dx}[e^u] = e^u \cdot \frac{du}{dx}$$

12. Chain Rule (natural logarithm function)

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot \frac{du}{dx}$$

13. Chain Rule (trigonometric functions)

$$\frac{d}{dx}[\sin u] = \cos u \cdot \frac{du}{dx} \qquad \frac{d}{dx}[\cos u] = -\sin u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}[\tan u] = \sec^2 u \cdot \frac{du}{dx} \qquad \frac{d}{dx}[\csc u] = -\csc u \cot u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}[\sec u] = \sec u \tan u \cdot \frac{du}{dx} \quad \frac{d}{dx}[\cot u] = -\csc^2 u \cdot \frac{du}{dx}$$

Derivative Properties:

1. Constant Multiple Rule

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

2. Sum/Difference Rule

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

3. Product Rule

$$\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)]g(x) + f(x)\frac{d}{dx}[g(x)]$$

4. Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{(g(x))^2}$$

Important Limits:

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1$$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{\theta\to 0} \frac{\cos\theta - 1}{\theta} = 0$$

$$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$$

$$\lim_{x\to\infty} (1+\frac{1}{n})^n = e$$