

Your name(s):

DAY 2: BASIC CONCEPTS AND COUNTING  
SEC 1.1-1.2

## 1 First Exercises

1. There are 30 students in this class.
  - (a) How many standing arrangements are possible if all students are waiting in line at the coffee machine at 7:45AM?
  - (b) What is the probability that you are picked to answer a random question? Call this event  $X$ . What is  $P(X)$ ?
  - (c) What is the probability that your best friend in class is picked to answer the aforementioned random question? Call this event  $Y$ .
  - (d) You know we can only call on one person to answer a specific question. What is  $P(X \cap Y)$ ? What is  $P(X \cup Y)$ ?
2. The five numbers 1, 2, 3, 4, and 5 are written respectively on five disks of the same size and placed in a hat. Two disks are drawn without replacement from the hat and the numbers written on them are observed.
  - (a) List the possible outcomes of this experiment as unordered pairs of numbers.
  - (b) If each outcome is equally likely, determine which is more likely: that the sum of the two numbers is odd or that the sum of the two numbers is even. Justify your answer by calculating an appropriate probability.
3. Olin loves committees. If we assume that Olin has  $n = 300$  students, how many different committees of size  $r = 4$  are possible?
4. How many ways can 40 distinguishable balls be placed into 365 distinguishable bins, so that no bin has more than one ball?
5. How many five-card poker hands are possible from a standard deck of cards? How many of these hands have at least four diamonds? What is the probability of getting a poker hand with at least four diamonds?
6. A pediatrician wishes to recruit 5 couples, each of whom is expecting their first child, to participate in a new natural childbirth regimen. Suppose that the probability that a randomly selected couple agrees to participate is 0.2. What is the probability that she needs to ask exactly 15 couples before 5 are found who agree to participate?

## 2 Second Exercises

1. You are given that  $P(A \cup B) = 0.7$  and  $P(A \cup B') = 0.9$ . Find  $P(A)$ . Hint: Use the definition!

2. Every year the Olin student body (assume again 300 students) elects a slate of officers for CoRE. Suppose we consider four elected positions: P, VP, T, C. How many ways can these elected positions to be filled, assuming that any student can fill any one position?
3. Say you run an experiment in flipping a fair coin 10 times. What is the probability that you flip your 5th head on exactly the 10th flip? Explain with mathematical formulas (define events as necessary) and common sense.