Probability + Statistics

Post-Class Problems – Oct 26

1.2 3. a) $\underbrace{26 \cdot 26}_{\text{Two letters (26 possible English letters)}} \cdot \underbrace{10 \cdot 10 \cdot 10 \cdot 10}_{\text{Four digits (10 possible digits)}} = 6{,}760{,}000$

b) $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17{,}576{,}000$

9. a) $\binom{9}{3} \cdot 6! \cdot 3! = 362880$

- $\binom{9}{3}$: First, we need to figure out which spots are dem and which are rep, Choose 3 spots from the 9 and stick the reps. there, Fill in the 6 dems in the remaining 6 spots.
- $6!$: Total number of different ways to order the dems relative to each other
- $3!$: Total number of different ways to order the reps relative to each other

b) By similar logic: $\binom{9}{3} = 84$

c) $\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{Each candidate either gets a R or D (2 possibilities)}} = 2^9 = 512$

11. a) There are 13 possible card values to get 4 of a kind. Once you pick the card value, there are 48 cards left in the deck to take the 5th place in your hand.

$$\frac{13 \cdot 48}{\binom{52}{5}} = 0.00024$$

b) $13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{2} \div \binom{52}{5} = 0.00144$

- 13: 13 possible values for the 3-of-a-kind
- $\binom{4}{3}$: choose 3 of the four cards of the chosen value to be in your hand
- 12: 12 remaining values for the pair
- $\binom{4}{2}$: choose 2 cards of that value to be in your hand
- $\binom{52}{5}$: total possible hands

c) $13 \cdot \binom{4}{3} \binom{12}{2} \binom{4}{1} \binom{4}{1} \div \binom{52}{5} = 0.02113$

- 13: 13 possible values for the 3-of-a-kind
- $\binom{4}{3}$: choose 3 of the cards of that value to go in your hand
- $\binom{12}{2}$: choose 2 more values for the remaining cards
- $\binom{4}{1}\binom{4}{1}$: choose the suits of the remaining cards

d) $\binom{13}{2} \binom{4}{2} \binom{4}{2} \cdot 44 \div \binom{52}{5} = 0.04754$

- $\binom{13}{2}$: choose 2 values for the pairs
- $\binom{4}{2}\binom{4}{2}$: pick the two from each value to go in your hand
- 44: 44 cards remain

e) $13 \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4 \cdot 4 \cdot 4 \div \binom{52}{5} = 0.42257$

- 13, $\binom{4}{2}$: choose a value and pick two cards of that value
- $\binom{12}{3}$: choose 3 cards from the remaining 12 values
- $4 \cdot 4 \cdot 4$: and pick one of the four cards of each value

1.3

5. a) $\{RR, RW, WR, WW\}$

b) $P(RR \mid \text{red eyes}) = \frac{P(RR \text{ and red eyes})}{P(\text{red eyes})}$

$= \frac{1/4}{3/4} = \frac{1}{3}$

7. Possible draws = $\{(O_1, O_2), (O_1, B_1), (O_1, B_2), (O_2, O_1), (O_2, B_1), (O_2, B_2), (B_1, B_2), (B_1, O_1), (B_1, O_2), (B_2, B_1), (B_2, O_1), (B_2 O_2)\}$

10 possible draws where at least one is orange.

2 possible draws where both are orange

$\frac{2}{10} = 0.2$

13. a)

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

b) 8 possible combinations that sum to 7 or 11

$\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (5,6), (6,5)\}$ $\therefore$ 8/36

c) 11 possible ways to roll an 8 or 7
5 ways to roll an 8 $\therefore$ 5/11

d) There are 5 ways to roll an 8 $\therefore P(8) = \frac{5}{36}$

$P(8)\ P(8 \mid 7 \text{ or } 8) = \left(\frac{5}{36}\right)\left(\frac{5}{11}\right)$

e) Following similar logic as above:

$$\frac{8}{36} + 2\left[\left(\frac{5}{36}\right)\left(\frac{5}{11}\right) + \left(\frac{4}{36}\right)\left(\frac{4}{10}\right) + \left(\frac{3}{36}\right)\left(\frac{3}{9}\right)\right] = 0.49293$$