Post-Class Solutions Due 19 Feb

3.4

① a) $P(0 \le Z \le 0.87) = \Phi(0.87) - \Phi(0) = 0.8078 - 0.5000 = \boxed{0.3078}$

b) $P(-2.64 \le Z \le 0) = \Phi(0) - \Phi(-2.64) = 0.5000 - (1 - \Phi(2.64))$
$= 0.5000 - (1 - 0.9959) = \boxed{0.4959}$

c) $P(-2.13 \le Z \le -0.56) = (1 - \Phi(0.56)) - (1 - \Phi(-2.13))$
$= (1 - .7123) - (1 - .9834) = \boxed{0.2711}$

d) $P(|Z| > 1.39) = \cancel{P(-1.39 > Z < 1.39)} = \cancel{(1-\Phi(-1.39)) - \Phi}\ 2(1 - \Phi(1.39)) = \boxed{0.1646}$

P(z > 1.39 or z < −1.39) $= \cancel{\Phi(1.39) - (1 - \Phi(1.39)) = 0.9177 - (1 - 0.9177)} = \cancel{\boxed{0.8354}}$

e) $P(Z < 1.62) = 1 - \Phi(1.62) = 1 - 0.9474 = \boxed{0.0526}$

f) $P(|Z| > 1) = P(Z > 1 \text{ or } Z < -1)\ \cancel{2(}\ 2(1 - \Phi(1)) = 2(1 - 0.8413)$
$= \boxed{0.3174}$

g) $P(|Z| > 2) = P(Z > 2 \text{ or } Z < -2) = 2(1 - \Phi(2)) = 2(1 - 0.9772)$
$= \boxed{0.0456}$

h) $P(|Z| > 3) = P(Z > 3 \text{ or } Z < 3) = 2(1 - \Phi(3)) = 2(1 - 0.9987)$
$= \boxed{0.0026}$

③ X is $N(6, 25)$ $\mu = 6$, $\sigma = 5$ $Z = \frac{X - \mu}{\sigma}$

a) $P\left(\frac{6-6}{5} \le Z \le \frac{12-6}{5}\right) = P(0 \le Z \le 1.2) = \boxed{0.3849}$

b) $P\left(\frac{0-6}{5} \le Z \le \frac{8-6}{5}\right) = P(-1.2 \le Z \le 0.4) = \boxed{0.5403}$

c) $P\left(\frac{-2-6}{5} \le Z \le \frac{0-6}{5}\right) = P(-1.6 < Z < -1.2) = \boxed{0.0603}$

d) $P\left(Z > \frac{21-6}{5}\right) = P(Z > 3) = \boxed{0.0013}$

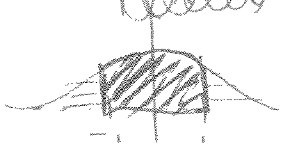
e) $P(|X - 6| > 5)$ $P(X - 6 \le 5)$ and $P(X - 6 \ge -5)$

$P(X \le 11)$ $P(X \ge 1)$

$P\left(Z \le \frac{11-6}{5}\right)$ $P\left(Z \ge \frac{1-6}{5}\right)$

$P(Z \le 1)$ and $P(Z \ge -1)$

$\Phi(1) - (1 - \Phi(1)) = \boxed{0.6826}$

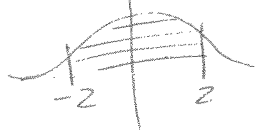


f) $P(|X-6|<10) \Rightarrow P(X-6<10)$ and $P(X-6>-10)$

$P(X<16)$ $\quad\quad$ $P(X>-4)$

$P\left(Z<\frac{16-6}{5}\right)$ $\quad\quad$ $P\left(Z>\frac{-4-6}{5}\right)$

$P(Z<2)$ $\quad\quad$ $P(Z>-2)$



$= \boxed{0.9544}$

g) $P(|X-6|<15) = \boxed{0.9974}$ $\quad$ h) $P(|X-6|<12.41) = \boxed{0.9900}$

(9) X is $N(7,4)$ $\mu=7, \sigma=2$

$P[15.364 \le (X-7)^2 \le 20.096]$

$= P[\sqrt{15.364} \le X-7 \le \sqrt{20.096}] + P[\sqrt{15.364} \le 7-X \le \sqrt{20.096}]$

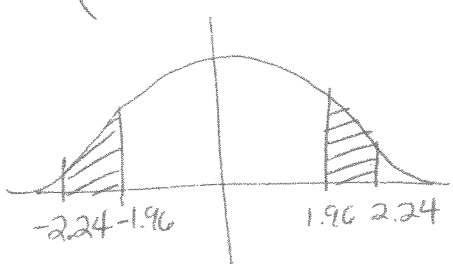
$= P[\sqrt{15.364}+7 \le X \le \sqrt{20.096}+7] + P[7-\sqrt{15.364} \ge X \ge 7-\sqrt{20.096}]$

$= P\left[\frac{\sqrt{15.364}}{2} \le Z \le \frac{\sqrt{20.096}}{2}\right] + P\left[\frac{-\sqrt{15.364}}{2} \ge Z \ge \frac{-\sqrt{20.096}}{2}\right]$

$= P(1.96 \le Z \le 2.24) + P(-1.96 \ge Z \ge -2.24)$

$= 2[\Phi(2.24) - \Phi(1.96)] = 2(.9875 - .9750)$

$= \boxed{0.025}$



(11) X is $N(75,100)$ $\mu=75$ $\sigma=10$

$P(X>85 \mid X>80) = P\left(Z>\frac{85-75}{10} \,\middle|\, Z>\frac{80-75}{10}\right) = P(Z>1 \mid Z>0.5)$

$= \frac{P(Z>1 \cap Z>0.5)}{P(Z>.5)} = \boxed{0.514}$

3.5

⑤ $f(x;\theta) = e^{-(x-\theta)}$ or $e^{\theta - x}$ $\quad \theta < x < \infty$

$L(\theta) = (e^{\theta - x_1})(e^{\theta - x_2}) \dots (e^{\theta - x_n})$

To maximize $L(\theta)$, we want to maximize θ due to the exponents.

However, since $\theta < x$, it ($\nearrow L(\theta)$) can at largest be equal to the smallest value of x.

$$\boxed{\hat{\theta} = \min(X_1, X_2, \dots, X_n)}$$

⑦ $\bar{X} \sim N(\theta, \theta^2/n)$ $\quad \hat{\theta} = \bar{X}$ $\quad \therefore N(\bar{X}, \bar{X}^2/n)$

95% confidence interval $= \mu \pm 2\sigma$

$\mu = \bar{X}$

$\sigma^2 = \dfrac{\bar{X}^2}{n}$ $\quad = \left[\bar{X} - \dfrac{2\bar{X}}{\sqrt{n}}, \bar{X} + \dfrac{2\bar{X}}{\sqrt{n}}\right]$ factored: $\boxed{\left[\bar{X}(1 - 2/\sqrt{n}), \bar{X}(1 + 2/\sqrt{n})\right]}$

$\sigma = \dfrac{\bar{X}}{n}$

alternate solution: 95% CI $-2 \le z \le 2$

$$z = \frac{\bar{X} - \theta}{\theta/\sqrt{n}} = \frac{\bar{X}}{\theta/\sqrt{n}} - \frac{\theta}{\theta/\sqrt{n}} = \frac{\bar{X}\sqrt{n}}{\theta} - \sqrt{n} = \sqrt{n}\left(\frac{\bar{X}}{\theta} - 1\right)$$

$$-2 \le \sqrt{n}\left(\frac{\bar{X}}{\theta} - 1\right) \le 2$$

$$-2/\sqrt{n} \le \frac{\bar{X}}{\theta} - 1 \le 2/\sqrt{n}$$

$$-2/\sqrt{n} + 1 \le \frac{\bar{X}}{\theta} \le 2/\sqrt{n} + 1$$

invert it

$$\frac{1}{1 - 2/\sqrt{n}} \ge \frac{\theta}{\bar{X}} \ge \frac{1}{1 + 2/\sqrt{n}}$$

$$\frac{\bar{X}}{1 + 2/\sqrt{n}} \le \theta \le \frac{\bar{X}}{1 - 2/\sqrt{n}}$$

$$\boxed{\left[\frac{\bar{X}}{1 + 2/\sqrt{n}}, \frac{\bar{X}}{1 - 2/\sqrt{n}}\right]}$$

Both answers are acceptable.