Probability & Statistics

Post-Class Problems Assigned 2/19/10 due 2/23/10
Sec 3.6 # 11, 13
Sec 3.7 # 3, 9, 11

3.6 11) $\mu=30, \sigma=3, n=100$

$P(\bar{X}>29.5) = P\left(Z > \frac{29.5-\mu}{\sigma/\sqrt{n}}\right)$

$= P\left(Z > \frac{29.5-30}{3/\sqrt{100}}\right)$

$= P\left(Z > \frac{5}{3}\right)$

$= 0.9522$

3.6 13) $Y = X_1 + X_2 + X_3 + X_4$ where X_1, X_2, X_3, X_4 have gamma dist $\Theta = 2, \alpha = 4$

$\mu_X = \alpha\Theta = 2 \cdot 4$

$= 8$

$\sigma_X^2 = \alpha\Theta^2 = 4 \cdot 2^2$

$= 16$

$P(Y \le 35) = P(\bar{X} \le 8.75)$ → $\bar{X}$ will have a normal dist because it is a sample mean & we're using C.L.T.

$P(\bar{X} \le 8.75) = P\left(Z \le \frac{8.75-\mu_{\bar{X}}}{\sigma/\sqrt{n}}\right)$

$= P\left(Z \le \frac{8.75-8}{\sqrt{16}/\sqrt{4}}\right)$

$= P(Z \le 0.375)$

$= 0.64462$

Comparing to gamma distribution w/

$\alpha = 4+4+4+4 = 16$

$\Theta = 2$

Using an online calculator $P(X \le 35) = 0.6725$ which shows the normal distribution to be a good approx

3.7 3) $p = 0.60, \quad n = 864$

$P(496 \le X \le 548) = P\left(\frac{496-np}{\sqrt{np(1-p)}} \le Z \le \frac{548-np}{\sqrt{np(1-p)}}\right)$

$= P\left(\frac{496-518.4}{14.4} \le Z \le \frac{548-518.4}{14.4}\right)$

$= P(-1.55 \le Z \le 2.05)$

$= \Phi(2.05) - \Phi(1.56)$

$= .9803 - .0594$

$= .9214$

Note: $P(.574 < \hat{p} < .634) = .9258$ for $b(.6, 864)$

3.7 9) $\lambda = 49$

$$P(45 < X < 60) = P\left(\frac{45.5-\mu}{\sigma} \le Z \le \frac{59.5-\mu}{\sigma}\right)$$
$$= P\left(\frac{45.5-49}{\sqrt{49}} \le Z \le \frac{59.5-49}{\sqrt{49}}\right)$$
$$= P(-\tfrac{1}{2} \le Z \le 1.51)$$
$$= .9345 - .3085$$
$$= 0.6260$$

3.7 11) $\lambda = \frac{2}{3}, n = 30 \rightarrow \lambda_x = 30 \cdot \frac{2}{3} = 20$

a) $P(15 < \sum_{i=1}^{30} X_i \le 22) = P\left(\frac{15.5-\lambda_x}{\sqrt{\lambda_x}} \le Z \le \frac{22.5-\lambda_x}{\sqrt{\lambda_x}}\right)$
$$= P\left(\frac{15.5-20}{\sqrt{20}} \le Z \le \frac{22.5-20}{\sqrt{20}}\right)$$
$$= P(-1.006 \le Z \le 0.559)$$
$$= .7123 - .1587$$
$$= 0.5536$$

b) $P(21 \le \sum_{i=1}^{30} X_i < 27) = P\left(\frac{21-\lambda}{\sqrt{\lambda}} \le Z \le \frac{26.5-\lambda}{\sqrt{\lambda}}\right)$
$$= P\left(\frac{21-20}{\sqrt{20}} \le Z \le \frac{26.5-20}{\sqrt{20}}\right)$$
$$= P(0.2236 \le Z \le 1.4534)$$
$$= .9625 - .5871$$
$$= 0.3754$$