Probability & Statistics

Pre-Class Problems 2/19/10
3.6 # 4, 6, 10
3.7 # 2, 6, 8

3.6 4) $f(x) = 1 - \frac{x}{2}$, $0 \le x \le 2$ → take sample $n = 18$

a) $\mu = \int_0^2 x f(x)\,dx$
$= \int_0^2 x(1 - \frac{x}{2})\,dx$
$= \int_0^2 x - \frac{1}{2}x^2\,dx$
$= \left[\frac{1}{2}x^2 - \frac{1}{6}x^3\right]_0^2$
$\mu = \frac{2}{3}$

$\sigma^2 = \int_0^2 x^2 f(x)\,dx - \mu^2$
$= \int_0^2 x^2(1 - \frac{x}{2})\,dx - (\frac{2}{3})^3$
$= \int_0^2 x^2 - \frac{1}{2}x^3\,dx - \frac{4}{9}$
$= \left[\frac{1}{3}x^3 - \frac{1}{8}x^4\right]_0^2$
$\sigma^2 = \frac{2}{9}$

b) $P(\frac{2}{3} \le \bar{X} \le \frac{5}{6})$ for a sample of $n = 18$
$= P\left(\frac{\frac{2}{3} - \mu}{\sigma/\sqrt{n}} \le \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \le \frac{\frac{5}{6} - \mu}{\sigma/\sqrt{n}}\right)$
$= P\left(\frac{\frac{2}{3} - \frac{2}{3}}{\sqrt{\frac{2}{9}}/\sqrt{18}} \le Z \le \frac{\frac{5}{6} - \frac{2}{3}}{\sqrt{\frac{2}{9}}/\sqrt{18}}\right)$
$= P(0 \le z \le 1.5)$
$= 0.4332$ (Answer comes from table)

3.6 6) a) $E(\bar{X}) = E(X) = \mu = 24.43$.
b) $Var(\bar{X}) = \frac{Var(X)}{n} = \frac{2.2}{30} = 0.0733$
c) $P(24.17 \le X \le 24.82)$
$= P\left(\frac{24.17 - \mu}{\sigma(\bar{X})} \le z \le \frac{24.82 - \mu}{\sigma(\bar{X})}\right)$
$= P\left(\frac{24.17 - 24.43}{.0733} \le z \le \frac{24.82 - 24.43}{.0733}\right)$
$= P(-0.96 \le z \le 1.44)$
$= 0.7566$

3.6 10) $\mu = 2000$, $\sigma = 500$, $n = 25$

$$P(\bar{X} > 2050) = P\left(z > \frac{2050 - 2000}{500/\sqrt{25}}\right)$$

$$= P(z > 0.5)$$
$$= 0.3085$$

3.7 2) $p = 0.20$ are left-handed/ambidextrous ("success in binomial dis

a) $P(2 \leq X \leq 9)$ for $b(25, 0.20)$

$$= P(X \leq 8) - P(X \leq 2)$$
$$= 0.9532 - 0.0982$$
$$= 0.8550$$

b) $P(2 < X < 9)$ using central limit thm

$$= P(2.5 \leq X \leq 8.5)$$

$$= P\left(\frac{2.5 - np}{\sqrt{np(1-p)}} \leq \frac{X - \mu}{\sigma} \leq \frac{8.5 - np}{\sqrt{np(1-p)}}\right)$$

$$\approx P\left(\frac{2.5 - 5}{\sqrt{25 \cdot .2 \cdot .8}} \leq z \leq \frac{8.5 - 5}{\sqrt{25 \cdot .2 \cdot .8}}\right)$$

$$\approx P(-1.25 \leq z \leq 1.75)$$
$$= 0.8543$$

3.7 6) $p = 0.7$, $n = 84$

$$P(X \leq 52) = P\left(z \leq \frac{52 - np}{\sqrt{np(1-p)}}\right)$$
$$= P\left(z \leq \frac{52 - 84 \cdot 0.7}{\sqrt{84(0.7)(0.3)}}\right)$$
$$= P(z \leq -1.5)$$
$$= 0.06668$$

3.7 8) $N(21.37, 0.16)$

a) $P(X < 20.857) = P\left(z < \frac{20.857 - 21.37}{\sqrt{0.16}}\right)$

$$= P(z < -1.282) = 0.10$$

b) The distribution of Y is $b(100, 0.10)$ ← came from a)

$$P(Y \leq 5) \approx P\left(z \leq \frac{5.5 - np}{\sqrt{np(1-p)}}\right)$$

$$\approx P\left(z \leq \frac{5.5 - 100 \cdot 0.1}{\sqrt{100 \cdot 0.1 \cdot 0.9}}\right)$$
$$\approx P(z \leq -1.50)$$
$$= 0.06668$$

3.7 8) Continued

c) $P(21.31 \leq \bar{X} \leq 21.39) = P\left(\frac{21.31 - 21.37}{\frac{0.4}{\sqrt{100}}} \leq z \leq \frac{21.39 - 21.37}{0.4/\sqrt{100}}\right)$

$= P(-1.50 \leq z \leq 0.50)$

$= 0.6247$