Probability & Statistics

Preclass Problems 2/23/10
4.1 #2,4,6
4.2# 2,8,12

4.1 2) $Y = X_1 + X_2$ w/ χ^2 distribution $\chi^2(14)$
X_1 has χ^2 distribution $\chi^2(3)$

a) $\mu_Y = \mu_1 + \mu_2$
$14 = 3 + \mu_2$
$\mu_2 = 11$
Guess that X_2 has χ^2 distribution $\chi^2(11)$
b) $P(3.053 < X_2 < 24.72) = 0.990 - 0.010$ (see table IV)
$= 0.98$

4.1 4) 3 Steps:

(Assume normal & independent)		μ	σ
	1)	6	2
	2)	4	2
	3)	5	3

$Y = X_1 + X_2 + X_3$
$\mu_Y = \mu_{x_1} + \mu_{x_2} + \mu_{x_3}$
$\mu_Y = 6 + 4 + 5$
$\mu_Y = 15$
$\sigma_Y^2 = \sigma_{x_1}^2 + \sigma_{x_2}^2 + \sigma_{x_3}^2$
$\sigma_Y^2 = (2)^2 + (2)^2 + (3)^2$
$\sigma_Y^2 = 17$

Y has a normal distribution $N(15, 17)$
$P(Y < 20) = P\left(\frac{Y-15}{\sqrt{17}} < \frac{20-15}{\sqrt{17}}\right)$
$= \Phi(1.213) = 0.8845$

4.1 6) Let $\sigma_x^2 = \sigma_y^2 = \sigma^2$ which is unknown
a) $\bar{X}$ is $N(\mu_x, \sigma^2/n)$
$\bar{Y}$ is $N(\mu_y, \sigma^2/m)$
Let $\bar{Q} = \bar{X} - \bar{Y}$ (Meghan likes the letter Q)
$\mu_Q = \mu_x - \mu_y$
$\sigma_Q^2 = \frac{\sigma^2}{n} + \frac{\sigma^2}{m} = \sigma^2\left(\frac{1}{n} + \frac{1}{m}\right)$
$\bar{Q}$ is $N(\mu_x - \mu_y, \sigma^2(\frac{1}{n} + \frac{1}{m}))$

4.1 (6) continued

b) $\frac{(n-1)S_x^2}{\sigma^2}$ is $\chi^2(n-1)$

$\frac{(m-1)S_y^2}{\sigma^2}$ is $\chi^2(m-1)$

Since X and y are independent

$\frac{(n-1)S_x^2}{\sigma^2} + \frac{(m-1)S_y^2}{\sigma^2}$ is $\chi^2(n-1+m-1)$

$[(n-1)S_x^2 + (m-1)S_y^2]/\sigma^2$ is $\chi^2(n+m-2)$

c) $$\frac{\dfrac{\bar{X}-\bar{Y}-(\mu_x-\mu_y)}{\sqrt{\sigma^2(\frac{1}{n}+\frac{1}{m})}}}{\sqrt{\dfrac{[n-1]S_x^2+[m-1]S_y^2]/\sigma^2}{n+m-2}}} = \frac{\dfrac{\bar{Q}-\mu_Q}{\sqrt{\sigma_Q^2}}}{\sqrt{\dfrac{\chi^2(n+m-2)}{n+m-2}}} = T_Q$$

Since Q has n+m ~~eleme~~ samples, this T is a distribution (student's) for Q w/ n+m-2 degrees of free

note: you can factor out σ^2

$$\frac{\dfrac{\bar{X}-\bar{Y}-(\mu_x-\mu_y)}{\cancel{\sigma}\sqrt{\frac{1}{n}+\frac{1}{m}}}}{\cancel{\sigma}\sqrt{\dfrac{[n-1]S_x^2+[m-1]S_y^2}{n+m-2}}} = \frac{\dfrac{\bar{X}-\bar{Y}-(\mu_x-\mu_y)}{\sqrt{\frac{1}{n}+\frac{1}{m}}}}{\sqrt{\dfrac{[n-1]S_x^2+[m-1]S_y^2}{n-m-2}}}$$

4.2 2) a) $\bar{X} = 273.04$

$S_x^2 = 315554$

b) since $n=25$ and $S_x^2 = 3155.4$ (use table 4 w/ 25 d.of.)

$\sigma^2 : \left[\frac{24 S_x^2}{\chi^2_{.05}}, \frac{24 S_x^2}{\chi^2_{.95}}\right]$

$\sigma^2 = \left[\frac{24(31554)}{36.42}, \frac{24(3155.4)}{13.85}\right]$

$\sigma^2 = [2079.43, 5468.08]$

c) $\sigma = \sqrt{\sigma^2} = [\sqrt{2079.43}, \sqrt{5468.08}]$

$= [45.60, 73.95]$

d) yes, assumption of normality seems valid after drawing a q-q plot

4.2 8) a) $\bar{X} = 3.243$ is an estimate of μ

b) $S^2 = 0.2372$ " " " " σ^2

$S = 0.487$ " " " " σ

c) $\mu = \left[\bar{X} \pm Z_{\alpha/2} \frac{S}{\sqrt{n}}\right]$, $\alpha = 0.10$ for a 90% confidence interval

$\mu = \left[3.243 \pm 1.796 \cdot \frac{0.487}{\sqrt{12}}\right]$

$\mu : [2.991, 3.495]$

4.2 12) $\mu_x - \mu_y = \left[(\bar{X} - \bar{Y}) \pm T_{\alpha/2} \sqrt{\frac{(n-1)S_x^2 + (m-1)S_y^2}{n+m-2}\left(\frac{1}{n} + \frac{1}{m}\right)}\right]$

$= \left[(65.7 - 68.2) \pm 2.485 \sqrt{\frac{11 \cdot 16 + 14 \cdot 9}{25}\left(\frac{1}{12} + \frac{1}{15}\right)}\right]$

$= [-5.485, 0.845]$