DAY 7 : INCLASS SOLS

1) X = DISTANCE FROM A

$$f(x)=\begin{cases}\frac{1}{B-A} & 0\le x\le (B-A)\\ 0 & \text{o.w.}\end{cases}$$

$$P\left(X\le \frac{(B+A)}{2}\right) = \int_0^{(B+A)/2}\frac{1}{B-A}dx = \frac{1}{B-A}x\Big|_0^{(B+A)/2} = \frac{(B+A)}{2}\cdot\frac{1}{B-A}=\frac{1}{2}$$

$$P\left(X\ge \frac{3(B-A)}{4}\right) = \int_{3(B-A)/4}^{(B-A)}\frac{1}{B-A}dx = \frac{1}{B-A}\left((B-A)-\frac{3(B-A)}{4}\right)=\frac{1}{4}$$

$$P(\text{PAST MIDPT}) = P\left(X>\frac{(B-A)}{2}\right)=\frac{1}{2}=p \quad (\text{BINOMIAL})$$

Y = NO. THAT LAND PAST

$$P(Y=1)=\binom{3}{1}p^1q^2 = 3\left(\frac{1}{2}\right)^3=\frac{3}{8}$$

2) $$f(x)=\begin{cases}\frac{1}{1000}e^{-x/1000} & x\ge 0\\ 0 & \text{o.w.}\end{cases}$$

case ① $x<0$ $\quad F(x)=\int^x 0\,dt=0$

case ② $0\le x$ $\quad F(x)=\int_0^x\frac{1}{1000}e^{-t/1000}dt$

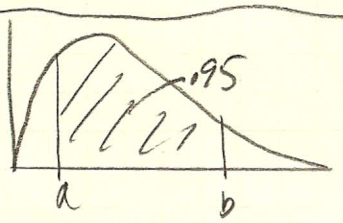
$$= -e^{-t/1000}\Big|_0^x = 1-e^{-x/1000}$$

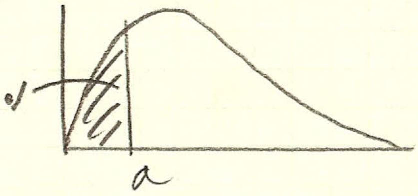
SO $$F(x)=\begin{cases}1-e^{-x/1000} & x\ge 0\\ 0 & \text{o.w.}\end{cases}$$

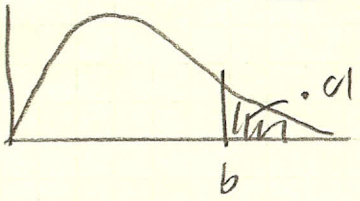
$$P(X>2000\mid X>1000)=\frac{P(1000\le X\le 2000)}{P(X>1000)}=\frac{F(2000)-F(1000)}{1-F(1000)}$$

$$=\frac{(1-e^{-2})-(1-e^{-1})}{1-e^{-1}} = \frac{e^{-1}-e^{-2}}{1-e^{-1}} = \frac{.2325}{.6321} = .3678$$

6)







$a=\chi^2_{.975}=3.247$ $\qquad$ $a=\chi^2_{.90}$ $\qquad$ $b=\chi^2_{.01}$

$b=\chi^2_{.025}=20.48$ $\qquad$ $=4.865$ $\qquad$ $=23.4$

3) $\int_0^\infty \frac{1}{\alpha} m x^{m-1} e^{-x^m/\alpha} dx \longrightarrow$ Let $u = x^m/\alpha$, $\frac{du}{dx} = \frac{mx^{m-1}}{\alpha}$, $du = \frac{1}{\alpha} m x^{m-1} dx$ $\rightarrow = \int e^{-u} du = -e^{-u} = -e^{-x^m/\alpha}\Big|_0^\infty$

$= 0 + e^0 = 1$

$\int_0^{\pi_{.5}} f(x)dx = \frac{1}{2} \Rightarrow -e^{-\frac{x^m}{\alpha}}\Big|_0^{\pi_{.5}} = 1 - e^{-\frac{\pi_{.5}^m}{\alpha}} = \frac{1}{2} \Rightarrow e^{-\frac{\pi_{.5}^m}{\alpha}} = .5$

$-\frac{\pi_{.5}^m}{\alpha} = \ln(.5) \Rightarrow \pi_{.5} = \sqrt[m]{-\alpha \ln(.5)}$

4) $\int_0^1 x(12x^2(1-x))dx = 12\int_0^1 x^3 - x^4 dx = 12\left(\frac{x^4}{4} - \frac{x^5}{5}\Big|_0^1\right) = 12\left(\frac{5}{20} - \frac{4}{20}\right)$

$= 12/20 = 3/5 = .6$

60%.

want $P(X > .4 \mid X \leq .6) = \frac{P(.4 \leq X \leq .6)}{P(X \leq .6)} = \frac{12\int_{.4}^{.6}(x^3 - x^4)dx}{12\int_0^{.6}(x^3-x^4)dx}$

$= \left(\frac{x^4}{4} - \frac{x^5}{5}\right)\Big|_{.4}^{.6} \Big/ \left(\frac{x^4}{4} - \frac{x^5}{5}\right)\Big|_0^{.6} = \ldots$

5) $\int_{-\infty}^{x} \frac{1}{\pi}\frac{1}{1+t^2}dt = \frac{1}{\pi}\tan^{-1}(t)\Big|_{-\infty}^{x} = \frac{1}{\pi}\tan^{-1}x - \frac{1}{\pi}\left(\frac{-\pi}{2}\right)$

$= \frac{1}{\pi}\tan^{-1}x + \frac{1}{2}$

1ST QUARTILE — SET $= .25$ $\quad \frac{1}{\pi}\tan^{-1}x + \frac{1}{2} = \frac{1}{4} \Rightarrow \tan^{-1}x = -\frac{\pi}{4}$

$x = -.6657$

2ND QUARTILE — SET $= .5$ $\quad \frac{1}{\pi}\tan^{-1}x + \frac{1}{2} = \frac{1}{2} \Rightarrow \tan^{-1}x = 0$

$x = 0$

3RD QUARTILE — SET $= .75$ $\quad \frac{1}{\pi}\tan^{-1}x + \frac{1}{2} = \frac{3}{4} \Rightarrow \tan^{-1}x = \frac{\pi}{4}$

$x = .6657.$

TOPS
35502