In-class Problems
8-Nov-2009

1.

a. $P\left(X \le \frac{B-A}{2}\right) = 0.5$

$P\left(X \ge \frac{3}{4}\cdot(B-A)\right) = 0.25$

b. If there are 3 parachutists, each have a 50% chance of landing past the midpoint.

$P(y=1) = \binom{3}{1}\cdot .5^2 \cdot .5^1 = \frac{3}{8}$

2.

$f(t) = 0 \qquad t<0$

$f(t) = \frac{1}{1{,}000}\cdot e^{-\frac{t}{1000}} \qquad t \ge 0$ → expected lifetime = 1000 hours, exponential distribution

a. $F(x) = \int_{-\infty}^{x} \frac{1}{1000}\cdot e^{-\frac{t}{1000}}\, dt$

If $x \ge 0$

$= 1 - e^{-\frac{t}{1000}}$

If $x < 0$: $\int_{-\infty}^{x} 0 \, dt = 0$

$$P(X \geq 2000 \mid X \geq 1000) = \frac{P(X \geq 2000)}{P(X \geq 1000)}$$

$$= \frac{1-F(2000)}{1-F(1000)}$$

$$= \frac{e^{-\frac{2000}{1000}}}{e^{-\frac{1000}{1000}}}$$

$$= \boxed{0.368}$$

3. a. Let $X = X_1 - X_2$ → gap between; we want to make sure the gap is positive.

$X_1 = N(\mu_1 = 30.25,\ \sigma_1 = .06)$
$X_2 = N(\mu_2 = 30,\ \sigma_2 = .05)$

$$X = N\left(\mu = \mu_1 - \mu_2,\ \sigma = \sqrt{\sigma_1^2 + \sigma_2^2}\right)$$

$$= N(0.25,\ .0781)$$

$$P(Y<0) = P\left(\frac{x-\mu}{\sigma} < \frac{0-0.25}{0.0781}\right) = P(Z \leq -3.2) = .0007$$

b. $$P(.1 < y < .35) = P\left(\frac{.1-.25}{.0781} \leq Z \leq \frac{.35-.25}{.0781}\right)$$

$$= P(-1.92 \leq Z \leq 1.28)$$

$$= 0.8997 - 0.0274 = \boxed{0.8723}$$

C. Binomial distribution

$n = 6$, $p = .8723$, $1-p = .1277$

$$P(X \geq 4) = \binom{6}{4} \cdot .8723^4 \cdot .1277^2 + \binom{6}{5} \cdot .8723^5 \cdot .1277 + \binom{6}{6} \cdot .8723^6$$

$$= \boxed{0.969}$$

4.

$$\mu = E[X] = \int_0^1 x \cdot f(x)\, dx$$

$$= \int_0^1 x \cdot 12x^2(1-x)\, dx$$

$$= 12 \int_0^1 x^3 - x^4\, dx$$

$$= 12 \quad \frac{1}{4}x^4\Big|_0^1 - \frac{1}{5}x^5\Big|_0^1$$

$$= \boxed{0.6}$$

probability it cannot be sold and it's less than mean impurity

$$P(X > 0.4 \mid X < 0.6) = \frac{P(0.4 \leq X \leq 0.6)}{P(X \leq 0.6)}$$

$$= \frac{\int_{0.4}^{0.6} f(x)\, dx}{\int_0^{.6} f(x)\, dx} = \frac{12 \cdot \frac{1}{3}x^3\Big|_{.4}^{.6} - \frac{1}{4}x^4\Big|_{.4}^{.6}}{12 \cdot \frac{1}{3}x^3\Big|_0^{.6} - \frac{1}{4}x^4\Big|_0^{.6}}$$

$$= \frac{0.296}{0.4752} = \boxed{0.623}$$