DAY 11 – MINI-LECTURE

SEC (4.1)

① THE SUM OF χ^2 RVS IS ALSO A χ^2 RV. ADD THE DEGREES OF FREEDOM.

$$\Rightarrow Z = \frac{X-\mu}{\sigma} \sim N(0,1)$$

Z^2 IS χ^2

② THM (4.1-2) – A WEIGHTED SUM OF INDEP. NORMAL RVS IS ALSO A NORMAL RV

$X_i \sim N(\mu_i, \sigma_i^2)$ FOR $i = 1, 2, \dots, n$

$$Y = \sum_{i=1}^{n} a_i X_i \sim N(\mu_Y, \sigma_Y^2) \qquad \mu_Y = \sum_{i=1}^{n} a_i \mu_i$$

$$\sigma_Y^2 = \sum_{i=1}^{n} a_i^2 \sigma_i^2$$

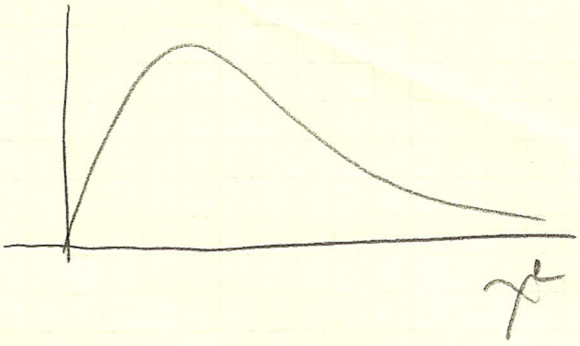
IF $a_i = \frac{1}{n}\ \forall i$, THEN

$$Y = \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

③ THE STATISTIC $\frac{(n-1)S^2}{\sigma^2}$ IS A χ^2 RV W/ $n-1$ DOF

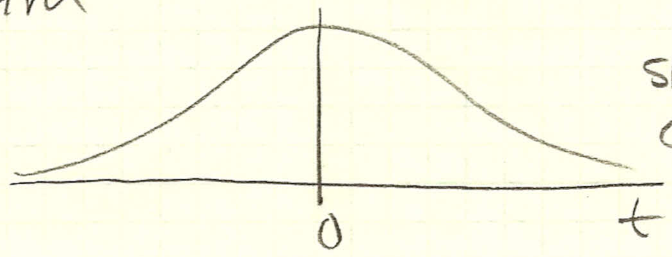
S^2 = SAMPLE VAR FROM RS $X_1, X_2, \dots, X_n$

σ^2 = POP. VAR.



(4) T-STATISTIC

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t\text{-DIST. w/ } n-1 \text{ D.O.F.}$$



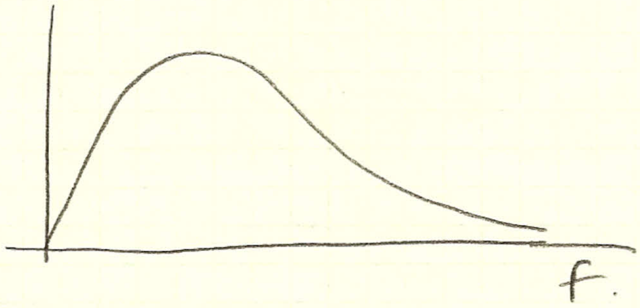
SIMILAR TO Z DIST
ONLY MORE VARIABILITY

(5) F-STATISTIC

$$\frac{S_x^2/\sigma_x^2}{S_y^2/\sigma_y^2} \sim F\text{-DIST. w/ } \begin{matrix} n-1 \text{ NUMERATOR} \\ m-1 \text{ DENOMINATOR} \end{matrix} \text{ DOFS.}$$

S_x^2 IS SAMPLE VAR FROM $X_1, \dots, X_n$ — σ_x^2 POP VAR FOR X

S_y^2 IS SAMPLE VAR FROM $Y_1, \dots, Y_m$ — σ_y^2 POP VAR FOR Y



NOTE: SUMMARY OF STATS.

$$z = \frac{x-\mu}{\sigma} \quad \text{OR} \quad z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$$

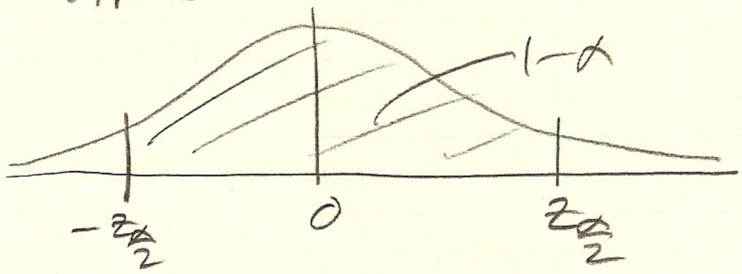
$$t = \frac{\bar{X}-\mu}{S/\sqrt{n}}$$

$$\chi^2 = (n-1)\frac{S^2}{\sigma^2} \qquad F = \frac{S_x^2/\sigma_x^2}{S_y^2/\sigma_y^2}$$

EACH STAT. IS USED TO CONSTRUCT A CONF. INTERVAL OR PERFORM A HYP. TEST.

EX. FIND <u>INTERVAL ESTIMATE</u> FOR POP. MEAN μ.
USE $\bar{X}$!

$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ IS RELATED STATISTIC



$$\underbrace{P(-z_{\frac{\alpha}{2}} \le Z \le z_{\frac{\alpha}{2}})}_{\downarrow} = 1-\alpha$$

$$-z_{\frac{\alpha}{2}} \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le z_{\frac{\alpha}{2}}$$

SOLVE FOR μ

$$\bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$$

SO $P(\bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}) = 1-\alpha$

SO $(\bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}})$

IS CALLED <u>100%(1-α) CONF. INTERVAL FOR μ</u>

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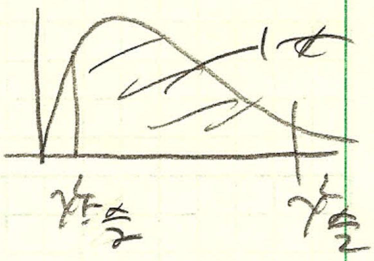
SBE(4.2)

CONSTRUCT CI FOR σ^2. USE $\chi^2 = (n-1)s^2/\sigma^2$

$$\underbrace{P(\chi^2_{1-\frac{\alpha}{2}} \le \chi^2 \le \chi^2_{\frac{\alpha}{2}})}_{\downarrow} = 1-\alpha$$



$$\chi^2_{1-\frac{\alpha}{2}} \le \frac{(n-1)s^2}{\sigma^2} \le \chi^2_{\frac{\alpha}{2}}$$

SOLVE FOR σ^2

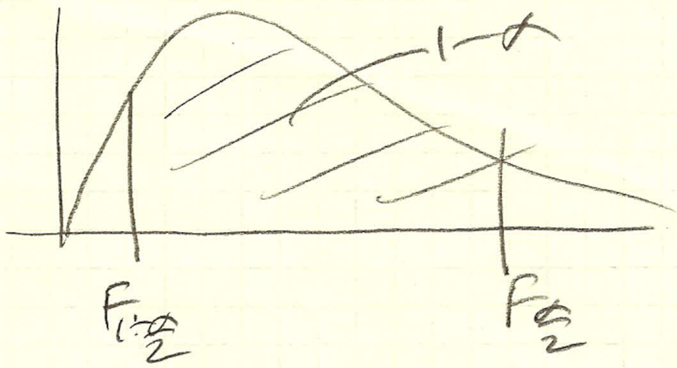
$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}$$

SO 100%(1−α) CI FOR σ^2 IS

$$\left(\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}}, \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}\right)$$

TO GENERATE CI FOR σ, JUST TAKE $\sqrt{\quad}$

NOW, CI FOR RATIO OF VARIANCES $\frac{\sigma_X^2}{\sigma_Y^2}$ IS GENERATED IN SIMILAR WAY



NOW $F_{1-\frac{\alpha}{2}}(m-1, n-1) = 1/F_{\frac{\alpha}{2}}(n-1, m-1)$

$$\underbrace{P(F_{1-\frac{\alpha}{2}} \le F \le F_{\frac{\alpha}{2}})}_{\downarrow} = 1-\alpha$$

$$F = \frac{S_Y^2/\sigma_Y^2 \quad m-1 \text{ DOF}}{S_X^2/\sigma_X^2 \quad n-1 \text{ DOF}}$$

$$F_{1-\frac{\alpha}{2}}(m-1,n-1) \le \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \le F_{\frac{\alpha}{2}}(m-1,n-1)$$

SOLVE FOR $\frac{\sigma_X^2}{\sigma_Y^2}$

$$\frac{1}{F_{\frac{\alpha}{2}}(n-1,m-1)} \frac{S_X^2}{S_Y^2} \le \frac{\sigma_X^2}{\sigma_Y^2} \le F_{\frac{\alpha}{2}}(m-1,n-1) \frac{S_X^2}{S_Y^2}$$

SO $100\%(1-\alpha)$ CONF INTERVAL FOR $\frac{\sigma_X^2}{\sigma_Y^2}$ IS

$$\left(\frac{1}{F_{\frac{\alpha}{2}}(n-1,m-1)} \frac{S_X^2}{S_Y^2} , F_{\frac{\alpha}{2}}(m-1,n-1) \frac{S_X^2}{S_Y^2} \right)$$

CAN CREATE CI FOR μ (σ UNKNOWN BASED ON t)
SEE PP 162-163

$$P(-t_{\frac{\alpha}{2}} \le t \le t_{\frac{\alpha}{2}}) = 1-\alpha \qquad t = \frac{\bar{X}-\mu}{S/\sqrt{n}}$$

SOLVE FOR μ

$100\%(1-\alpha)$ CI IS

$$\left(\bar{X} - t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} , \bar{X} + t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}\right)$$

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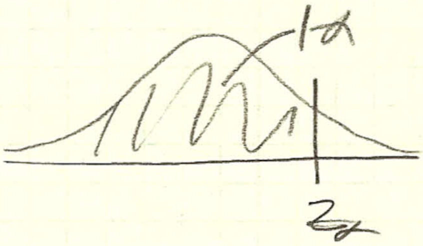
SO FAR, EACH CI HAS BEEN "TWO-SIDED".
WE CAN ALSO GENERATE "ONE-SIDED" CIS

EX. $\underbrace{P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le z_\alpha\right) = 1-\alpha}$



$\swarrow$ SOLVE FOR μ

$$P\left(\bar{X} - z_\alpha \frac{\sigma}{\sqrt{n}} \le \mu\right) = 1-\alpha$$

SO $(\underbrace{\bar{X} - z_\alpha \frac{\sigma}{\sqrt{n}}}_{\text{LOWER CONF. BOUND.}}, \infty)$ IS ONE-SIDED CI FOR μ

EX. $P\left(-z_\alpha \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\right) = 1-\alpha$

$\swarrow$ SOLVE FOR μ

$$P\left(\mu \le \bar{X} + z_\alpha \frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

SO, $(-\infty, \underbrace{\bar{X} + z_\alpha \frac{\sigma}{\sqrt{n}}}_{\text{UPPER CONF. BOUND.}})$ IS ONE-SIDED CI FOR μ

<u>NOTE</u> ALL OTHER CIS CAN BE GENERATED AS ONE SIDED.

WHEN COMPARING TWO POPS TOGETHER, NEED TO USE Z OR t STATISTICS

$$Z = \frac{(\bar{X}-\bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim N(0,1)$$

USE AS LONG AS σ_X^2, σ_Y^2 ARE KNOWN.

IF UNKNOWN, USE

$$t = \frac{(\bar{X}-\bar{Y}) - (\mu_X - \mu_Y)}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}}$$ (SEE p. 165)

$$S_p^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2}$$ ← POOLED VARIANCE

RESPECTIVE CONF. INTERVALS GENERATED ARE

$$(\bar{X}-\bar{Y}) \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}$$

$$(\bar{X}-\bar{Y}) \pm t_{\frac{\alpha}{2}}\, S_p\sqrt{\frac{1}{n} + \frac{1}{m}}$$

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