Your name(s):

DAY 7: MAXIMUM LIKELIHOOD ESTIMATORS, CENTRAL LIMIT THEOREM
SEC 3.5-3.6

1. A certain type of electrical component has a lifetime Y (in hours) with probability density function given by

$$f(x) = \begin{cases} \frac{1}{\theta^2} y e^{-y/\theta} & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What is the likelihood function for an independent random sample $Y_1, Y_2, \ldots Y_n$ from this distribution? Write an expression for the log-likelihood function.

   (b) Derive the maximum likelihood estimator $\hat{\theta}$ for parameter θ.

2. Let $X_1, X_2, \ldots, X_n$ denote an independent random sample from the following probability distribution

$$f(x) = \begin{cases} (\theta + 1)x^{\theta} & 0 \leq x \leq 1 \ , \ \theta > -1 \\ 0 & \text{otherwise} \end{cases}$$

   Find the maximum likelihood estimator for the parameter θ.

3. Suppose $T_1, T_2, \ldots T_n$ are independent times until failure of n electronic components. Each failure time has the identical exponential distribution with expected time until failure of $\mu = 1000$. Let $\bar{T}$ be the average time until failure of n such randomly selected machines. Determine an approximate distribution for $\bar{T}$ and make a sketch of it on the the same graph as a sketch of the distribution of the time until failure of a single machine.

4. Shear strength measurements for spot welds have been found to have standard deviation 10 psi. If 100 test welds are to be measured, what is the approximate probability that the sample mean will be within 1 psi of the true population mean?

5. Achievement test scores of all high school seniors in a state have mean 60 and variance 64. A random sample of $n = 100$ students from one large high school had a sample mean score of 58. Is there evidence to suggest that this high school is inferior with respect to the others in the state?