

1) $S = \{1, 2, 3, 4, 5, 6\}$

a)

X	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

 EQUALLY LIKELY

b) $E(X) = \sum_x x P(X=x) = \frac{1}{6}(1+2+3+4+5+6) = \frac{1}{6}(21) = 3.5$

c) $E(X^2) = \sum_x x^2 P(X=x) = \frac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2) = \frac{1}{6}(91) = 15.1\bar{6}$

d) $\sigma^2 = E(X^2) - \mu^2 = \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{91}{6} - \frac{49}{4} = \frac{364-294}{24} = \frac{70}{24} = 2.91\bar{6}$

2) $P(T+|OP) = .997$ $P(OP) = .6$ $P(T+) = P(T+|OP)P(OP) +$
 $P(T+|VS) = .9995$ $P(VS) = .27$ $P(T+|VS)P(VS) +$
 $P(T+|CC) = .897$ $P(CC) = .13$ $P(T+|CC)P(CC)$

NOW, FIND $P(CC|T+)$

$= \frac{P(T+|CC)P(CC)}{P(T+)}$

$P(T+) = \frac{(.897)(.13)}{(.9846)} = .1184$

$= (.997)(.6) + (.9995)(.27) + (.897)(.13)$
 $= .9846$

3) A \equiv ASKED QUESTION "HOW FAST WAS CAR MOVING WHEN IT PASSED BARN?"

$P(A) = .5$ \leftarrow HALF SUBJECTS ASKED

$P(\bar{A}) = .5$

B \equiv ASKED QUESTION "IS THERE A BARN IN FILM?" AND SAID YES.

$P(B|A) = .17$ $P(B|\bar{A}) = .03$

a) $P(B) = P(B|\bar{A})P(\bar{A}) + P(B|A)P(A) = (.03)(.5) + (.17)(.5) = .1$

b) IS $P(B|A) = P(B)$?

$.17 \neq .1$ NO, NOT INDEPENDENT!

4) BINOMIAL PROBLEM

$$p = \frac{8}{20} = 0.4 \text{ (PROB. SERVICE ORDER FOR USER PRINTER)}$$

$$p = 1 - q = \frac{12}{20} = 0.6 \text{ (" " " " INJECT ")} \leftarrow \text{SUCCESS!}$$

$$n = 5 \text{ (SAMPLE SIZE)}$$

$$\textcircled{a} P(X=2) = \binom{5}{2} (0.6)^2 (0.4)^3 = \frac{5!}{2!3!} (0.36)(0.064) = 0.576$$

$$\textcircled{b} P(X=x) = \binom{5}{x} (0.6)^x (0.4)^{5-x}$$

5) NEGATIVE BINOMIAL - X = NO. GAMES PLAYED TO DETERMINE WINNER

$$= 4, 5, 6, 7$$

$$r = 4 \text{ (NEED 4 WINS TO END SERIES)}$$

4 GAMES

AAAA
NNNN

5 GAMES

AAAJA
AANAA
AJAAA
NAAAA

6 GAMES

etc

7 GAMES

etc

$$\binom{4-1}{4-1} p^4 q^{4-4} = p^4 \Rightarrow \text{SINCE AAAA and NNNN ARE MUTUALLY EXCLUSIVE}$$

$$\binom{4-1}{4-1} q^4 p^{4-4} = q^4 \quad P(X=4) = P(AAAA) + P(NNNN) = p^4 + q^4 = \binom{4-1}{4-1} p^4 q^{4-4} + \binom{4-1}{4-1} q^4 p^{4-4}$$

EXTRAPOLATE \Rightarrow ARB. x

$$P(X=x) = \binom{x-1}{4-1} p^4 q^{x-4} + \binom{x-1}{4-1} q^4 p^{x-4}$$

$$\textcircled{b} E(X) = \sum_x x P(X=x) = \sum_x x \frac{(x-1)!}{3!(x-4)!} (p^4 q^{x-4} + q^4 p^{x-4})$$

$$= \sum_x \frac{x!}{x \cdot 3! (x-4)!} (p^4 q^{x-4} + q^4 p^{x-4})$$