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Probability and Statistics, Fall 2009, Quiz 3
Sections 2.3-2.5, 3.2-3.3, Special Discrete Distributions, Estimation,
Linear Functions of Independent Random Variables, Continuous
Probability Distributions, Special Continuous Probability Distributions

- No resources are allowed, except for a calculator/computer for basic arithmetic (including combinations and permutations); do not use any pre-programmed formulas.
- Write all answers in order on additional sheets of paper. Staple all pages together.
- Clearly explain your answers and show your work.
- There is a strict 90 minute limit for the quiz. Set an alarm. Aim for 60 minutes. Initial Quiz Download/View (Time and Date):
  End of Quiz (Time and Date):

1. A parking lot has two entrances. Cars arrive at entrance I on average of 3 per hour, while cars arrive at entrance II on average of 4 per hour. If the number of cars arriving at the two entrances are independent, what is the probability that a total of three cars will enter the parking lot in a given hour?
2. Over a long period of time, it is known that 13% of a company's audits of its financial records contain substantial errors. Beginning in November 2009, a CPA audits a series of this company's accounts. What is the probability that the first account containing substantial errors:
   (a) is the third one to be audited by the CPA?
   (b) will occur on or after the third audited account?
3. The lifetime of a certain brand of 100 watt light bulb follows an exponential distribution with an average service life of 1500 hours.
   (a) What is the probability that a bulb lasts more than 1000 hours?
   (b) Determine the *median* service life of a randomly selected 100 watt light bulb.
4. Let $X_1, X_2, \ldots, X_n$ denote an independent random sample, where each random variable X_i follows a Poisson probability distribution with parameter λ. Derive the maximum likelihood estimator $\hat{\lambda}$ for the parameter λ.
5. Patients with congestive heart failure typically have low *cardiac ejection fractions* (CEFs), which is the proportion of blood pumped out of the heart during a single beat. Let the RV Y denote the CEF of such patients, which has probability distribution given by

$$f(y) = \begin{cases} 30y^2(1-y)^2 & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Patients with acute congestive heart failure have CEFs below 25%. What is the probability that a randomly selected patient has acute congestive heart failure?
   (b) Ten patients admitted to a local hospital are examined for acute congestive heart failure. What is the likelihood that fewer than two have acute congestive heart failure?