Pre-Class Solutions for 2/5

2.4

② $L(p) = \prod_{i=1}^{n} \binom{x_i - 1}{r-1} p^r (1-p)^{x_i - r}$

$\ln L(p) = \ln\left[\prod_{i=1}^{n}\binom{x_i-1}{r-1}\right] nr\ln p + \left(\sum_{i=1}^{n} x_i - nr\right)\ln(1-p)$

$$\frac{d \ln L(p)}{dp} = \frac{nr}{p} - \frac{\sum_{i=1}^{n} x_i - nr}{1-p} = 0$$

$nr - nrp - p\sum_{i=1}^{n} x_i + nrp = 0$

$$\hat{p} = \frac{nr}{\sum_{i=1}^{n} x_i}$$

Note that $\hat{p}$ equals the # of successes divided by the number of Bernoulli trials.

④ a) $x=0 \to 17$ | $x=5 \to 28$
$x=1 \to 47$ | $x=6 \to 21$
$x=2 \to 63$ | $x=7 \to 11$
$x=3 \to 63$ | $x=8 \to 1$
$x=4 \to 49$

b) $\bar{x} = \frac{303}{100} = 3.03$

$s^2 = \frac{4141}{1300} = 3.193$

c) Poisson is best with $\lambda = 3.03$. → $\bar{x}$ and s^2 are about equal

Translated negative binomial with $p = \bar{x}/s^2$ and $r = 57$ also works. (since $\bar{x} < s^2$)

2.5

② $Var(Y) = Var(3X_2 - X_1)$

$$25 = 9\,Var(X_2) + (-1)^2 Var(X_1)$$

$$25 = 9(2) + k$$

$$25 = 18 + k \qquad k = 7$$

④ a) $\mu_Y = E(X_1)\,E(X_2)$ (due to independence)

$$\mu_Y = \mu_1 \mu_2$$

b) $Var(Y) = E[(X_1 X_2 - \mu_1\mu_2)^2]$

$$= E(X_1^2 X_2^2) - 2\mu_1\mu_2 E(X_1X_2) + \mu_1^2\mu_2^2$$

from part a

$$= (\sigma_1^2 + \mu_1^2)(\sigma_2^2 + \mu_2^2) - 2\mu_1\mu_2\,\mu_1\mu_2 + \mu_1^2\mu_2^2$$

$$\cancel{= \sigma_1^2\sigma_2^2 + \sigma_1^2\mu_2^2 + \sigma_2^2\mu_2^2 +}$$

$$= (\sigma_1^2 + \mu_1^2)(\sigma_2^2 + \mu_2^2) - \mu_1^2\mu_2^2$$

$$Var(Y) = \sigma_1^2\sigma_2^2 + \mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2$$