Block Print Your Name:

Probability and Statistics, Spring 2010, Final Exam
Chapters 1-4

- No resources are allowed, except for a calculator/computer for basic arithmetic; do not use any pre-programmed formulas.
- Explain your work!!!!
- Good luck!!!!

The following formulas might be helpful. If you use them, explain your choices.

$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

$T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$, $(n-1)$ degrees of freedom

$\chi^2 = (n-1)S^2/\sigma^2$, $(n-1)$ degrees of freedom

$Z = \frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\sigma_X^2/n+\sigma_Y^2/m}}$

$T = \frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\frac{(n-1)S_X^2+(m-1)S_Y^2}{n+m-2}(1/n+1/m)}}$, $(n+m-2)$ degrees of freedom

$F = \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2}$, $(m-1)$ numerator degrees of freedom and $(n-1)$ denominator degrees of freedom

Problem	Score
1	
2	
3	
4	
5	
6	
7	
TOTAL	

1. The following data have been compiled on drivers insured by the “Better Luck Next Time” Insurance Company. Note that it is a law that all drivers must have insurance.

Age of Driver	Proportion of All Drivers	Probability of Having an Accident
16-20	0.08	0.06
21-30	0.15	0.03
31-65	0.49	0.02
$\geq$ 66	0.28	0.04

(a) Find the probability that a randomly selected driver has an accident.

(b) If a randomly selected driver has an accident, calculate the probability that the driver was age 16-20.

2. Ella is a high school basketball player. She is a 70% free throw shooter, meaning her probability of successfully making a free throw on a single try is 0.70. We are interested in analyzing how many shot attempts it will take for Ella to successfully make exactly three shots.

   (a) Write down the functional form of the probability distribution $f(x)$ for this experiment. Indicate what x represents and explain all numbers/terms in your function.

   (b) Suppose Ella successfully makes her third free throw on her fifth shot attempt. What was the probability of that happening?

   (c) Given that Ella has successfully made her third free throw on her fifth shot attempt, what is the probability that Ella successfully makes her fifth free throw on her seventh shot attempt?

3. The IQs of applicants of a certain college are normally distributed with mean 115 and variance 144.

   (a) If the college requires an IQ of at least 95 for admission, what is the probability that a randomly selected student applicant will be rejected on this basis, regardless of their other qualifications?

   (b) If 300 students applied for admission, what is the probably that fewer than three were rejected for not meeting the IQ requirement?

4. An insurance company provides home insurance for its customers. The insured value Y (in hundreds of thousands of dollars) of a randomly selected home follows a continuous probability distribution with probability density function given by

$$f(y) = \begin{cases} \frac{3}{y^4} & y \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the mean μ and variance σ^2 for the random variable Y.

(b) A random sample of customers of size 5 is taken and the average insured value is computed. What is the probability that the average insured value is more than \$350,000?

5. The length of time between billing and receipt of payment was recorded for a random sample of 10 of a certified public accountant (CPA) firm's clients. The sample mean and standard deviation for the 10 accounts were 39.1 days and 17.3 days, respectively.

   (a) Find a 90% confidence interval for the mean time between billing and receipt of payment for all of the CPA firm's clients.

   (b) The billing department claims that, on average, clients tend to pay their bills within 30 days of billing. Is the claim valid? Explain your answer.

6. A researcher would like to test the hypothesis that mothers with low socio-economic status (SES) deliver babies whose birth weights are lower than normal. A sample of birth weights from 100 consecutive, full-term, live-born deliveries are obtained from a hospital in a low-SES area. The mean birth weight is found to be 115 oz with a standard deviation of 24 oz. It is known from nationwide surveys that the mean birth weight of all babies in the US is 120 oz. Define an appropriate null hypothesis and alternative hypothesis in order to perform a hypothesis test at the $\alpha = .01$ level, and then discuss the conclusions that can be drawn from your resulting analysis.

7. The claim that the variance of a normal population is $\sigma^2 = 25$ is to be rejected if the variance derived from a random sample of size 16 exceeds 50.967 or is less than 12.102. What is the probability that the claim is rejected, even though $\sigma^2 = 25$?