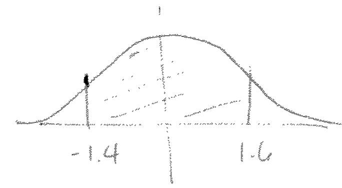
Solutions to Post-Class Problems Due 26 Feb

4.1

(3) $Y = X_1 - X_2 \qquad \mu_Y = \mu_1 - \mu_2 \qquad \sigma_Y^2 = \sigma_1^2 + \sigma_2^2 \qquad \sigma_Y = \sqrt{25} = 5$

$$\begin{aligned}
P(-10 < Y < 5) &= P\left(\frac{-10-\mu}{\sigma} < z < \frac{5-\mu}{\sigma}\right) \\
&= P\left(\frac{-10-(-3)}{5} < z < \frac{5-(-3)}{5}\right) \\
&= P(-1.4 < z < 1.6) \\
&= \Phi(1.6) - \Phi(-1.4) \\
&= \Phi(1.6) - (1 - \Phi(1.4)) \\
&= 0.9432 - (1 - 0.9192) = \boxed{0.8644}
\end{aligned}$$



(5) a) $P\left(-d < \frac{\bar{x}-\mu}{s/\sqrt{16}} < d\right) = 0.95 \qquad T = \frac{\bar{x}-\mu}{s/\sqrt{16}}$

$$\begin{aligned}
P(T<d) - P(T<-d) &= 0.95 \\
P(T<d) - (1 - P(T<d)) &= 0.95 \\
2P(T<d) - 1 &= 0.95 \\
P(T<d) &= 0.975
\end{aligned}$$

Use Table VI and notice that $r = n-1 = 16-1 = 15$

$\boxed{d = 2.131}$

b)
$$\begin{aligned}
&P\left(-2.131 < \frac{\bar{x}-\mu}{s/\sqrt{16}} < 2.131\right) = 0.95 \\
&P\left(\tfrac{s}{4}(-2.131) < \bar{x}-\mu < \tfrac{s}{4}(2.131)\right) = 0.95 \\
&P\left(\bar{x} - \tfrac{s}{4}(-2.131) > \mu > \bar{x} - \tfrac{s}{4}(2.131)\right) = 0.95 \\
&P\left(\bar{x} - \left(\tfrac{s}{4}\right)2.131 < \mu < \bar{x} + \left(\tfrac{s}{4}\right)2.131\right) = 0.95 \\
&P\left(u(\bar{x},s) < \mu < v(\bar{x},s)\right) = 0.95
\end{aligned}$$

$$\boxed{\begin{aligned} u(\bar{x},s) &= \bar{x} - \tfrac{s}{4}(2.131) \\ v(\bar{x},s) &= \bar{x} + \tfrac{s}{4}(2.131) \end{aligned}}$$

⑦ a) $\therefore P(-d<T<d)=0.90$

$$P(T<d)-P(T<-d)=0.90 \qquad r=m+n-2$$
$$P(T<d)-(1-P(T<d))=0.90 \qquad r=8+10-2$$
$$2P(T<d)-1=0.90 \qquad r=16$$
$$P(T<d)=0.95$$

Using Table VI, $\boxed{d=1.746}$

b) $$T=\frac{\dfrac{\bar{X}-\bar{Y}-(\mu_X-\mu_Y)}{\sqrt{\sigma^2(1/n+1/m)}}}{\sqrt{\dfrac{[(n-1)S_X^2+(m-1)S_Y^2]/\sigma^2}{n+m-2}}} \qquad T=\frac{\dfrac{\bar{X}-\bar{Y}-(\mu_X-\mu_Y)}{(1/8+1/10)}}{\sqrt{\dfrac{7S_X^2+9S_Y^2}{16}}} \quad \text{and } d=1.746$$

(from 4.1-6 (c) – notice that σ^2's cancel out)

$$\therefore P(-d<T<d)=P\left(\bar{X}-\bar{Y}-1.746\sqrt{\frac{7S_X^2+9S_Y^2}{16}}\sqrt{\frac{1}{8}+\frac{1}{10}}<T<\bar{X}-\bar{Y}+1.746\sqrt{\frac{7S_X^2+9S_Y^2}{16}}\sqrt{\frac{1}{8}+\frac{1}{10}}\right)$$

c) $$\left[7.3-6.4-1.746\sqrt{\frac{7(12.7)^2+9(10.3)^2}{16}}\sqrt{\frac{1}{8}+\frac{1}{10}},\ 7.3-6.4+1.746\sqrt{\frac{7(12.7)^2+9(10.3)^2}{16}}\sqrt{\frac{1}{8}+\frac{1}{10}}\right]$$

$[-8.5517, 10.3517]$

4.2

(3) $$\left[\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\chi^2_{\alpha/2}(n)}, \frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\chi^2_{1-\alpha/2}(n)} \right]$$

b) $\hat{\sigma} = s = \sqrt{var[x]} = \boxed{0.512}$

c) 95% one-sided CI: $r = n-1$

$[0, \bar{x} + t_\alpha(r)(\sigma/\sqrt{n})]$

10 observations $r = 10-1 = 9$

$= [0, 3.58 + t_{0.05}(9)(0.512/\sqrt{10})]$

lowest possible μ is 0

$\boxed{= [0, 3.877]}$

look in Table VI

(11) a) $\bar{x} = 25.475$

$s = 2.4935$

b) $[\bar{x} - t_\alpha(r)(\sigma/\sqrt{n}), \infty)$

$[25.475 - t_{0.01}(19)(2.4935/\sqrt{20}), \infty)$

$\boxed{[24.059, \infty)}$

(13) a) $s_p = \sqrt{\frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}}$ $\quad t_0 = t_{\alpha/2}(n+m-2)$ 26

$n = 12$

$\bar{x} = 712.25$

$s_x = 173.08$

$\bar{y} = 705.437$

$s_y = 141.711$

$m = 16$

CI: $\left[\bar{x} - \bar{y} - t_0 s_p \sqrt{\frac{1}{n} + \frac{1}{m}}, \bar{x} - \bar{y} + t_0 s_p \sqrt{\frac{1}{n} + \frac{1}{m}} \right]$

$\boxed{[-115.480, 129.105]}$

$s_p = \sqrt{\frac{(11)(173.08)^2 + (15)(141.711)^2}{26}} = 155.757$

$t_0 = t_{0.025}(26)$

$t_0 = 2.056$

(on back)

c) Since 0 (i.e. no difference) is within the confidence interval, we cannot conclude that there is a difference in butterfat production.

CI: $6.8125 \pm 2.056(155.757)\sqrt{\frac{1}{12}+\frac{1}{16}}$

$[-115.479, 129.105]$