Probability & Statistics

Pre-class problems 3/2
Sec 4.4 #6, 10
Sec 4.5 #2, 4abce, 6ab

4.4 6) $H_0: \sigma^2 = 140^2$, $H_1 = \sigma^2 > 140^2$

a) $\alpha = 0.05$, $n = 25$, 1-sided test, chi-square test stat $\left[\frac{(n-1)s^2}{\sigma^2}\right]$

$\chi^2_{.05}(24) = 36.42$

$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{24s^2}{140^2}$

b) $s^2 \approx 23.827$

$\chi^2 = \frac{24(23.827)}{140^2} = 29.18 < 36.42$ → fail to reject H_0

c) Upper bound for μ (98% conf)

$\bar{X} = 668$

$z_\alpha = 2.04$

$\bar{X} + z_\alpha \frac{\sigma}{\sqrt{n}} = 668 + 2.04 \cdot \frac{140}{\sqrt{25}}$

$= 725$

4.4 10) $H_0: p = 0.40$, $H_1: p > 0.40$, $n = 25$

y is number of first serves that are good

a) $\alpha = P(Y \geq 13; p = 0.40) = 1 - P(Y \leq 12)$

$= 1 - 0.8462$

$= 0.1538$

b) $\beta = P(Y \leq 12; p = 0.60) = 1 - P(Y \leq 12, p = 0.60)$

$= 1 - 0.8462$

$= 0.1538$

c) $p = P(Y \geq 15; p = 0.40) = 1 - P(Y \leq 14, p = 0.40)$

$= 1 - .9656$

$= 0.0344$

4.5 2) X: concentration of particles in Melbourne, Y: Houston

$n_x = 13$, $m_y = 16$, assume equal pop. variance

$H_0: \mu_x = \mu_y$; $H_1: \mu_x < \mu_y$

a) $$t = \frac{(\bar{X} - \bar{Y}) - 0}{S_p\sqrt{\frac{1}{n} + \frac{1}{m}}} < -t_{.05}(n+m-2)$$

$$S_p = \sqrt{\frac{(n-1)S_x^2 + (m-1)S_y^2}{n+m-2}} \quad \cancel{t_{.025}(27)}$$

$$t = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{12S_x^2 + 15S_y^2}{27}}\sqrt{\frac{1}{13} + \frac{1}{16}}} < -t_{.05}(27)$$

$$t = \frac{(\bar{X} - \bar{Y})}{\sqrt{\frac{12S_x^2 + 15S_y^2}{27} \cdot \left(\frac{1}{13} + \frac{1}{16}\right)}} < -1.703$$

b) $$t = \frac{(72.9 - 81.7)}{\sqrt{\left(\frac{12 \cdot 25.6^2}{27} + \frac{15 \cdot 28.3^2}{27}\right)\left(\frac{1}{13} + \frac{1}{16}\right)}} = -.869 > -1.703$$

Fail to reject H_0

c) $0.10 < \text{p-value} < 0.25$

d) $F = \frac{S_x^2}{S_y^2} < F_{0.025}(12, 15)$

$F = \frac{25.6^2}{28.3^2} = 0.818 < 2.96$

$F = \frac{S_y^2}{S_x^2} = \frac{28.3^2}{25.6^2} = 1.22 < 3.18 = F_{0.025}(15, 12)$

} assumption is valid

4.5 4) $X: N(\mu_x, \sigma_x^2) \rightarrow$ force req. to pull stud 3

$Y: N(\mu_y, \sigma_y^2) \rightarrow$ " " " " " 4

$m = n = 10$ observations, assume $\sigma_x^2 = \sigma_y^2$

$H_0: \mu_x - \mu_y = 0$; $H_1: \mu_x - \mu_y \neq 0$ (two-sided), $\alpha = 0.05$

a) $z_{\alpha/2} = \pm 1.96$

$$|z| = \frac{|(\bar{X} - \bar{Y}) - 0|}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}} \geq 1.96$$

b) $$|z| = \frac{|131.5 - 144.2|}{\sqrt{\frac{14.08^2}{10} + \frac{12.26^2}{10}}} = 2.15 > 1.96$$

Reject H_0

c) $0.01 \leq \text{p-value} \leq 0.05$

e) $F = \frac{S_y^2}{S_x^2} = \frac{12.26^2}{14.08^2} = 0.758 < 4.03 = F_{0.025}(9,9)$

$F = \frac{S_x^2}{S_y^2} = \frac{14.08^2}{12.26^2} = 1.319 < 4.03 = F_{0.025}(9,9)$

4.5 6) $H_0: \mu_x = \mu_y$, $H_1: \mu_x \neq \mu_y$, assume $\sigma_x^2 = \sigma_y^2$ → two sided test

$\bar{x} = 80.464$ where $X: N(\mu_x, \sigma_x^2)$ $s_x^2 = .0316$ $n = 10$

$\bar{y} = 80.378$ where $y = N(\mu_y, \sigma_y^2)$ $s_y^2 = .0915$ $m = 10$

a) Let $\alpha = 0.05$ → $z_{\alpha/2} = \pm 1.96$

$$|z| = \frac{|\bar{x} - \bar{y}|}{\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}} \geq 1.96$$

$$\cancel{|z| = \frac{|80.464 - 80.378|}{\sqrt{\frac{.0316}{10} + \frac{.0915}{10}}} = 0.775 < 1.96}$$

Fail to reject null hypothesis $0.20 \leq p \leq 0.25$

b) $F = \frac{s_x^2}{s_y^2} = \frac{.0316}{.0915} = .345 < 4.03 = F_{.025}(9,9)$

$F = \frac{s_y^2}{s_x^2} = \frac{.0915}{.0316} = 2.896 < 4.03 = F_{.025}(9,9)$