PRELIM EX.

DAY 6: INCLASS SOLS

1) $f(x_i) = \binom{n}{x_i} p^{x_i} (1-p)^{n-x_i}$ FOR $i=1,\dots,k$

$f(x_1,x_2,\dots,x_k) = f(x_1)f(x_2)\cdots f(x_k) = \prod_{i=1}^{k} f(x_i) = \prod_{i=1}^{k} \binom{n}{x_i} p^{x_i}(1-p)^{n-x_i}$

$= L(p) = \left(\prod_{i=1}^{k}\binom{n}{x_i}\right) p^{\Sigma x_i}(1-p)^{\Sigma n-x_i}$

TAKE NATURAL LOG!

$\ell(p) = \ln(L(p)) = \ln\left(\prod_{i=1}^{k}\binom{n}{x_i}\right) + (\Sigma x_i)\ln p + (\Sigma n-x_i)\ln(1-p)$

$\frac{d\ell}{dp} = 0 + \frac{k\bar{x}}{p} + \frac{(kn-k\bar{x})}{1-p}(-1) = 0 \qquad \frac{k\bar{x}}{p} = \frac{k(n-\bar{x})}{1-p}$

$\Rightarrow k\bar{x}(1-p) = k(n-\bar{x})p \Rightarrow k\bar{x} = knp \Rightarrow \hat{p} = \frac{\bar{x}}{n}$

$k\bar{x} - \cancel{k\bar{x}p} = knp - \cancel{k\bar{x}p}$

CHECK TO SEE IF MAX! $\frac{d\ell^2}{dp^2} = -\frac{k\bar{x}}{p^2} + \frac{(k(n-\bar{x}))(-1)}{(1-p)^2}\Bigg|_{p=\frac{\bar{x}}{n}}$

$= -\frac{k\bar{x}}{\bar{x}^2}n^2 + \frac{-kn+k\bar{x}}{(1-\frac{\bar{x}}{n})^2} \Rightarrow$ SINCE $\bar{x} \le n$ ALWAYS

$< 0 \quad \therefore \hat{p} = \frac{\bar{x}}{n}$ IS M.L.E.

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2) $f(x_i) = \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$ FOR $i=1,2,\dots,k$

$f(x_1,x_2,\dots,x_k) = f(x_1)f(x_2)\cdots f(x_k) = \prod_{i=1}^{k} f(x_i) = \prod_{i=1}^{k} e^{-\lambda}\lambda^{x_i}/x_i!$

TAKE NATURAL LOG! $= e^{-k\lambda}\lambda^{\Sigma x_i}/\left(\prod_{i=1}^{k} x_i!\right) = L(\lambda)$

$\ell(\lambda) = \ln(L(\lambda)) = -k\lambda + (\Sigma x_i)\ln\lambda - \ln\left(\prod_{i=1}^{k} x_i!\right)$

$\frac{d\ell}{d\lambda} = -k + \frac{k\bar{x}}{\lambda} = 0 \Rightarrow k = \frac{k\bar{x}}{\lambda} \Rightarrow \lambda = \bar{x}$

CHECK TO SEE IF MAX! $\frac{d\ell^2}{d\lambda^2} = -\frac{k\bar{x}}{\lambda^2}\Bigg|_{\lambda=\bar{x}} = -\frac{k\bar{x}}{\bar{x}^2} = -\frac{k}{\bar{x}} < 0$

SO $\hat{\lambda} = \bar{x}$ IS M.L.E. FOR λ

TOPS. 35502

3) a) GEOMETRIC. SUCCESS = HEADS $X_1 = X_2$ = # TRIAL ON WHICH 1ST SUCCESS OCCURS

$f(X_1) = (1-p)^{X_1-1}p \quad ; \quad f(X_2) = (1-p)^{X_2-1}p$

b) SINCE THE COINS ARE PHYSICALLY DIFFERENT AND THE TOSSES DON'T AFFECT EACH OTHER, WE SHOULD HAVE INDEPENDENT RVS

$f(X_1, X_2) = f(X_1)f(X_2) = (1-p)^{(X_1+X_2)-2}p^2$

c) $E(Y) = E(X_1 - X_2) = E(X_1) - E(X_2) = \frac{1}{p} - \frac{1}{p} = 0$

d) $V(Y) = V(X_1 - X_2) = V(X_1) + V(X_2) = \frac{(1-p)}{p^2} + \frac{(1-p)}{p^2} = \frac{2(1-p)}{p^2}$

TWICE AS MUCH VARIABILITY AS EITHER X_1 OR X_2 ALONE.

4) $n = 1234$ 864 FAVOR, 370 OPPOSE $\quad \hat{p} = \frac{864}{1234} \quad \hat{q} = \frac{370}{1234}$

CI IS $\hat{p} \pm 2\sqrt{\frac{\hat{p}\hat{q}}{n}} = .7 \pm 2\sqrt{\frac{(.7)(.3)}{1234}}$ → .6739, .726

$= (.6739, .726)$

2ND EX.

5) X = # ATTEMPTS TO SINK 1ST FREETHROW

$P(X=x) = (1-p)^{x-1}p \quad P(X=5) = (1-p)^4 p = L(p)$

$\ell(p) = \ln(L(p)) = 4\ln(1-p) + \ln p$

$\frac{d\ell}{dp} = \frac{4}{1-p}(-1) + \frac{1}{p} = 0 \Rightarrow 4p = 1-p \quad p = 1/5$
$5p = 1$

$\frac{d\ell^2}{dp^2} = \frac{4}{(1-p)^2}(-1) + \frac{-1}{p^2} < 0$ ALWAYS $\therefore \hat{p} = \frac{1}{5}$ IS MLE.

6) X = # ATTEMPTS BEFORE 3 SUCCESSES

$P(X=x) = \binom{6}{2}p^3(1-p)^4 = L(p) \quad \ell(p) = \ln\binom{6}{2} + 3\ln p + 4\ln(1-p)$

$\frac{d\ell}{dp} = \frac{3}{p} + \frac{4(-1)}{1-p} = 0 \quad 3/p = \frac{4}{1-p} \Rightarrow 3(1-p) = 4p \quad \hat{p} = \frac{3}{7}$
$3 = 7p$

$\frac{d^2\ell}{dp^2} = -\frac{3}{p^2} + \frac{-4}{(1-p)^2} < 0 \quad \therefore \hat{p} = \frac{3}{7}$ IS M.L.E

7) $\bar{X} = 5.82$ CI IS $\bar{X} \pm 2\frac{S}{\sqrt{n}} = 5.82 \pm \frac{\overbrace{2(.2)}^{.08}}{5}$

$S = .2$

$n = 25$ $= (5.74, 5.9)$ $= \begin{cases} 5.9 \\ 5.74 \end{cases}$

SINCE $\mu = 6 \notin$ CI, MACHINE IS FILLING LESS THAN 6 LBS PER BOX!