Your name(s):

DAY 4: BAYES RULE AND DISCRETE PROBABILITY DISTRIBUTIONS
SEC 1.5-2.1

1 First Exercises

1. Four technicians regularly make repairs when breakdowns occur on an automated production line. Janet, who services 20% of the breakdowns, makes an incomplete repair 1 time in 20. Tom, who services 60% of the breakdowns, makes an incomplete repair 1 time in 10. Georgia, who services 15% of the breakdowns, makes an incomplete repair 1 time in 10. Peter, who services 5% of the repairs, makes an incomplete repair 1 time in 20. For the next problem with the production line diagnosed as being due to an initial repair that was incomplete, what is the probability that this initial repair was made by Janet?

2. An E! team estimates that when it introduces its new product to the public, it will be very successful (VS) with a probability of 0.6, moderately successful with a probability of 0.3 (MS), and not successful (NS) with a probability of 0.1. The estimated yearly profit associated with being VS is $15mil; estimated yearly profit for MS is $5 mil; and estimated yearly loss for NS is $500,000. Let X be the yearly profit of the new product. Determine the PMF for X.

3. A second E! team is confident their product will always consistently earn $7 million per year. Find the expected profits for the first and second teams. Before you pick which team to back, you should consider the variability or possible deviation of the expected profits. Describe the variance for both teams' profits. What units are used? What alternative measure would give you a more easily interpreted measure of variability?

4. Suppose $O = \{1, 2, 3, \ldots\}$ and assign probabilities to the outcomes as follows:

$$p_k = \Pr(\text{Outcome is } k) = (1-p)^{k-1}p,\ 0 < p < 1,\ k = 1, 2, 3, \ldots.$$

   Verify that p_k is a PMF on the outcome space O. This probability distribution is known as the *geometric* PMF.

5. An insurance company issues life insurance policies in three separate categories: standard, preferred, and ultra-preferred. Of the companys policyholders, 50% are standard, 40% are preferred, and 10% are ultra-preferred. Each standard policyholder has probability 0.010 of dying in the next year, each preferred policyholder has probability 0.005 of dying in the next year, and each ultra-preferred policyholder has probability 0.001 of dying in the next year. What is the probability that a policyholder dies in the next year. Given that a policy holder dies in the next year, what is the probability that the deceased policyholder was ultra-preferred?

2 Second Exercises

1. In a gambling game, a person draws a single card from an ordinary 52-card playing deck. A person is paid $15 for drawing a jack or queen and $5 for drawing a king or ace. If they

draw any other card, they must pay \$$x$. Derive an expression for the expected gain/loss as a function of x. How large can x be for the game to be profitable to the person playing?

2. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease. Calculate the probability that a person has the disease given that the test indicates the presence of the disease.

3. Five balls, numbered 1, 2, 3, 4 and 5, are placed in an urn. Two balls are randomly selected without replacement from the five and their numbers are noted. Find the probability distribution for the *largest* of the two sampled numbers. Compute the mean and the variance.

4. Let X be $b(2, p)$ and Y be $b(4, p)$ (binomial RVs with different numbers of trials). If $P(X \geq 1) = \frac{5}{9}$, find $P(Y \geq 1)$.