

## DAY 4: IN CLASS SOLS

FIRST  
EX

$$\begin{aligned}
 1) \quad & P(J) = .2 \quad P(I|J) = .05 \quad P(I) = P(I|J)P(J) + P(I|T)P(T) + P(I|G)P(G) + P(I|P)P(P) \\
 & P(T) = .6 \quad P(I|T) = .1 \\
 & P(G) = .15 \quad P(I|G) = .1 \\
 & P(P) = .05 \quad P(I|P) = .05 \\
 & = (.05)(.2) + (.1)(.6) + (.1)(.15) + (.05)(.05) = .0875
 \end{aligned}$$

$$\text{NOW } P(J|I) = P(I|J)P(J)/P(I) = (.05)(.2)/.0875 = .1143$$

$$2) \quad \begin{array}{c|ccc} X & -500,000 & 5,000,000 & 15,000,000 \\ \hline P(X=x) & .1 & .3 & .6 \end{array}$$

$$3) \quad \begin{aligned} X_1 &\equiv \text{YEARLY PROFIT FROM TEAM 1} \\ X_2 &\equiv \text{ " " " TEAM 2} \end{aligned}$$

$$E(X_1) = \sum_x x P(X=x) = (-500,000)(.1) + 5,000,000(.3) + 15,000,000(.6) = \$10,450,000 \quad (10.45 \text{ MIL})$$

$$E(X_2) = \$7,000,000 \quad (7 \text{ MIL})$$

$$V(X_1) = E(X_1^2) - \mu_1^2 \quad \text{AND} \quad V(X_2) = 0 \quad (\text{NO VARIATION})$$

$$\begin{aligned}
 \hookrightarrow & \sum_x x^2 P(X=x) = (-500,000)^2(.1) + (5,000,000)^2(.3) + (15,000,000)^2(.6) \\
 & - (10,450,000)^2 \\
 & = 3.33225 \times 10^{13}
 \end{aligned}$$

$$SD = \sqrt{V(X_1)} = \$5,773,564.00 \quad \text{LARGE STD DEVIATION}$$

$$4) \quad \text{FIRST, } p_k = (1-p)^{k-1} p \geq 0 \text{ ALWAYS SINCE } 0 < p < 1 \quad \text{SET } (1-p) = q$$

$$\text{SECOND, } \sum_{k=1}^{\infty} p_k = \sum_{k=1}^{\infty} p(1-p)^{k-1} = p(1 + q + q^2 + q^3 + \dots)$$

$$\text{THIS IS A GEOMETRIC SERIES} = p \left( \frac{1}{1-q} \right) = \frac{p}{p} = 1$$

$\therefore$  IT IS A VALID P.M.F.

- 5)  $S \equiv$  STANDARD POLICY  $P(S) = .5$   $P(D|S) = .01$   
 $F \equiv$  PREFERRED "  $P(F) = .4$   $P(D|F) = .005$   
 $U \equiv$  ULTRA PREFERRED "  $P(U) = .1$   $P(D|U) = .001$   
 $D \equiv$  DISEASE IN NEXT IL

$$P(D) = P(D|S)P(S) + P(D|F)P(F) + P(D|U)P(U) = .0071$$

$$\text{Now, } P(U|D) = P(D|U)P(U)/P(D) = (.001)(.1)/.0071 = .0141$$

SECOND  
EX

- 1)  $X = \text{GAIN/LOSS}$

$$E(X) = \sum x p(x) = (15)\left(\frac{8}{52}\right) + 5\left(\frac{8}{52}\right) - X\left(\frac{36}{52}\right)$$

$$= \frac{160 - 36X}{52}$$

PROFITABLE  $\Rightarrow E(X) > 0$   $160 - 36X > 0$   $X < \frac{160}{36} = 4.4$   
 $160 > 36X$

- 2)  $Y = \text{POS. TEST RESULT}$

$D \equiv$  DISEASE PRESENT

$$P(Y|D) = .95 \quad P(D) = .01$$

$$P(Y|D') = .005 \quad P(D') = .99$$

$$\text{SO } P(Y) = P(Y|D)P(D) + P(Y|D')P(D') = .01445$$

$$\text{Now } P(D|Y) = P(Y|D)P(D)/P(Y) = (.95)(.01)/.01445 = .6574$$

- 3)

$$1, 2 \rightarrow 2$$

$$1, 3 \rightarrow 3$$

$$1, 4 \rightarrow 4$$

$$1, 5 \rightarrow 5$$

$$2, 3 \rightarrow 3$$

$$2, 4 \rightarrow 4$$

$$2, 5 \rightarrow 5$$

$$3, 4 \rightarrow 4$$

$$3, 5 \rightarrow 5$$

$$4, 5 \rightarrow 5$$

(CAN ALSO BE IN REVERSE ORDER)

20 TOTAL  
OUTCOMES

$X$	2	3	4	5
$P(X=x)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

$$\mu = E(X) = (2)\left(\frac{1}{10}\right) + (3)\left(\frac{2}{10}\right) + (4)\left(\frac{3}{10}\right) + (5)\left(\frac{4}{10}\right) = \frac{40}{10} = 4$$

$$E(X^2) = (2)^2\left(\frac{1}{10}\right) + (3)^2\left(\frac{2}{10}\right) + (4)^2\left(\frac{3}{10}\right) + (5)^2\left(\frac{4}{10}\right) = \frac{170}{10}$$

$$\sigma^2 = V(X) = \frac{170}{10} - \frac{1600}{100} = \frac{1700 - 1600}{100} = 1$$



4) SINCE  $P(X \geq 1) = \frac{5}{9}$ ,  $P(X=0) = \frac{4}{9}$

$$\Rightarrow \binom{2}{0} p^0 (1-p)^2 = \frac{4}{9} \Rightarrow (1-p)^2 = \frac{4}{9} \quad 1-p = \frac{2}{3} \quad p = \frac{1}{3}$$

NOW  $P(Y \geq 1) = 1 - P(Y=0)$

$$= 1 - \binom{4}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4 = 1 - \left(\frac{2}{3}\right)^4 = \frac{81}{81} - \frac{16}{81} = \frac{65}{81}$$