Probability + Statistics

Nov 16 Preclass Problems

$\boxed{4.1}$

2. a) $\mu_Y = \mu_1 + \mu_2$

$14 = 3 + \mu_2$

$11 = \mu_2$

$\chi^2(11)$

b) From Table IV, $0.99 - 0.01 = 0.98$

4. $Y = X_1 + X_2 + X_3$

$\mu_Y = 6 + 4 + 5$
$= 15$

$\sigma_Y^2 = 4 + 4 + 9$
$= 17$

$\therefore Y = N(15, 17)$

$P(Y < 20) = P\left(\frac{Y-15}{\sqrt{17}} < \frac{5}{\sqrt{17}}\right) = \Phi(1.213) = 0.8875$

6. a) $\bar{X} - \bar{Y}$ is the difference of two independent normal random variables w/ mean $\mu_X - \mu_Y$ and variance $\sigma^2/n + \sigma^2/m = \sigma^2(1/n + 1/m)$

$\therefore \bar{X} - \bar{Y} = N(\mu_X - \mu_Y, \sigma^2(1/n + 1/m))$

b) $(n-1)S_X^2/\sigma^2$ is $\chi^2(n-1)$ and $(m-1)S_Y^2/\sigma^2$ is $\chi^2(m-1)$ and they are independent, so $(n-1)S_X^2/\sigma^2 + (m-1)S_Y^2/\sigma^2$ is $\chi^2(n+m-1)$

c) From (a) the numerator of T is $N(0,1)$ and from (b) the denominator of T is the square root of a chi-square variable divided by its degrees of freedom. Since $(\bar{X}, \bar{Y})$ is independent of (S_X^2, S_Y^2), the numerator and denom of T are indep. By def'n, T has a Student's t distribution w/ n+m-2 degs of freedom

$\boxed{4.2}$

2) a) $\bar{x} = 273.04$ $\quad s^2 = 3155.54$

b) $\left[\frac{24(3155.54)}{36.42}, \frac{24(3155.54)}{13.85}\right] = [2079.43, 5468.08]$

c) $\left[\sqrt{2079.4333}, \sqrt{5468.08}\right]$

d) Yes, generate q-q plot

~~4~~8) a) $\bar{x} = 3.243$

b) $s^2 = 0.2372$, $s = 0.487$

c) $\left[3.243 - 1.796\left(\frac{0.487}{\sqrt{12}}\right), 3.243 + 1.796\left(\frac{0.487}{\sqrt{12}}\right)\right]$

$= [2.991, 3.495]$

12) $\left[65.7 - 68.2 \pm 2.485\sqrt{\frac{11(16)+14(9)}{25}}\sqrt{\frac{1}{12}+\frac{1}{15}}\right]$

$= [-5.845, 0.845]$