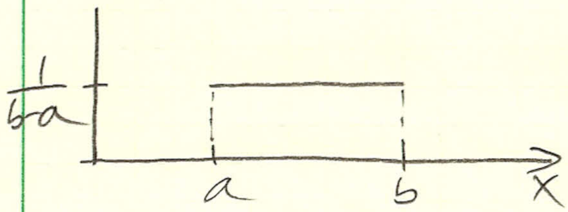
DAY 7 - CLASS NOTES

SEC(3.2)

GENERAL UNIFORM PDF $f(x)=\begin{cases}\frac{1}{b-a} & a\le x\le b\\ 0 & o.w.\end{cases}$



EX. CONSIDER $f(x)=c\sqrt{x}$ $\quad 0<x<4$

SINCE $\int_{-\infty}^{\infty} f(t)\,dx=1 \Rightarrow \cancel{\int_{-\infty}^{0} 0\,dx}^{\,0} + \int_{0}^{4} c\sqrt{x}\,dx + \cancel{\int_{4}^{\infty} 0\,dx}^{\,0} = 1$

$\Rightarrow c\frac{x^{3/2}}{3/2}\Big|_0^4 = \frac{2}{3}c\left(4^{3/2}-0^{3/2}\right) = \frac{2}{3}c\cdot 8 = 1 \qquad c=\frac{3}{16}$ ✓

REALLY, $f(x)=\begin{cases}0 & x\le 0\\ \frac{3}{16}\sqrt{x} & 0<x<4\\ 0 & x\ge 4\end{cases}$

WE CAN FIND C.D.F. (FUNCTION OF CUMULATIVE PROB)

<u>DEF</u> CDF $F(x)=\int_{-\infty}^{x} f(t)\,dt$ $\qquad F(x)=\begin{cases}0 & x\le 0\\ \frac{1}{8}x^{3/2} & 0<x<4\\ 1 & x\ge 4\end{cases}$

Case ① $x\le 0$ $\quad F(x)=\int_{-\infty}^{x} 0\,dt = 0$

Case ② $0<x<4$ $\quad F(x)=\int_{-\infty}^{0} 0\,dt + \int_{0}^{x}\frac{3}{16}\sqrt{t}\,dt = \frac{3}{16}\cdot\frac{2}{3}t^{3/2}\Big|_0^x$

$= \frac{1}{8}x^{3/2}$

Case ③ $x\ge 4$ $\quad F(x)=\int_{-\infty}^{0} 0\,dt^{\,0} + \int_{0}^{4}\frac{3}{16}\sqrt{t}\,dt + \int_{4}^{x} 0\,dt = 0 = 1$

DEF. X IS A CONT. RV W/ PDF $f(x)$

① POP. MEAN

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

② POP. VAR

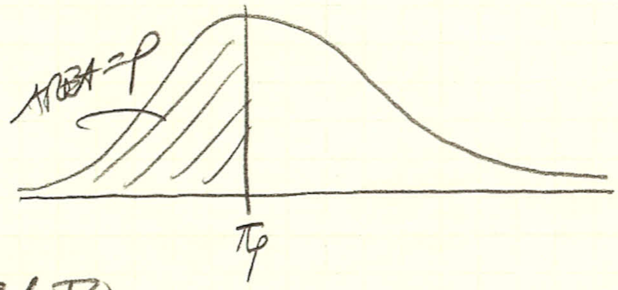
$$\sigma^2 = E((X-\mu)^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

$$= \left(\int_{-\infty}^{\infty} x^2 f(x)\,dx\right) - \mu^2$$

③ POP. S.D.

$$\sigma = \sqrt{\sigma^2}$$

DEF. PTH PERCENTILE



IF YOU KNOW CDF, EASY TO CALCULATE

$$p = \int_{-\infty}^{\pi_p} f(x)\,dx = F(\pi_p)$$

SEC(3.3)

USE CONT. UNIFORM DIST. TO SHOW

$$\mu = \frac{a+b}{2} \qquad \sigma^2 = \frac{(b-a)^2}{12}$$

DEF. X IS AN EXPONENTIAL RV IF

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & x \ge 0 \\ 0 & \text{ow} \end{cases}$$

WHAT ARE μ AND σ^2? (USE IBP)

DERIVE CDF Case ① $x < 0 \rightarrow F(x) = \int_{-\infty}^{x} 0\,dt$

Case ② $x \ge 0 \rightarrow F(x) = \int_{-\infty}^{0} 0\,dt + \int_{0}^{x} \frac{1}{\theta} e^{-t/\theta}\,dt$

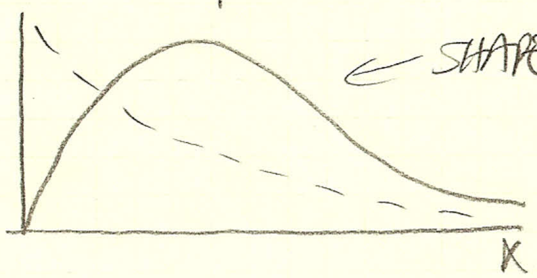
TO FIND MEDIAN, SOLVE $F(\pi_{.5}) = .5$ USING CDF

NOTE. "WAITING TIME" OR "TIME TO FAILURE" USUALLY MODELED AS EXPONENTIAL RV.

DEF. X IS A GAMMA RV IF

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\theta^{\alpha}} x^{\alpha-1} e^{-x/\theta} & x \ge 0 \\ 0 & \text{ow} \end{cases}$$

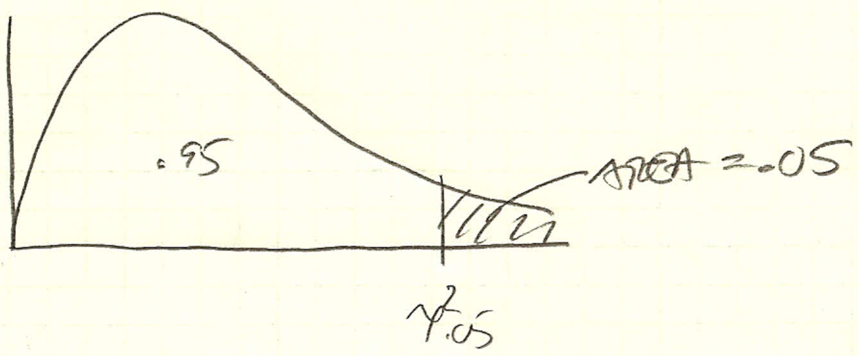
WHAT ARE μ AND σ^2?

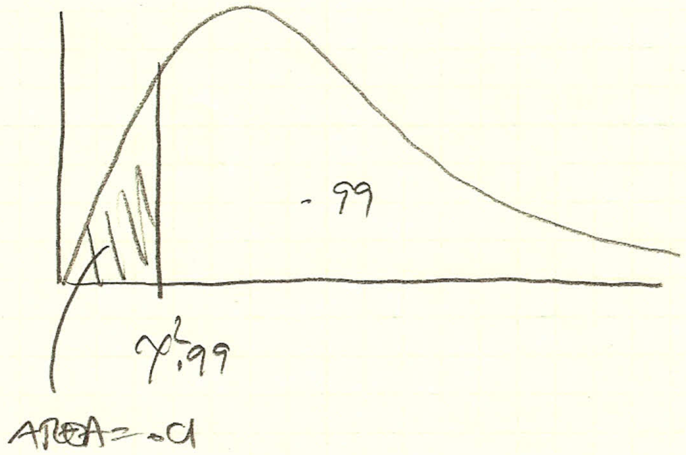


← SHAPE DEPENDS ON α, θ

SPECIAL CASE $\theta = 2$, $\alpha = \frac{r}{2}$ CALLED "CHI-SQUARE" (χ^2)

r CALLED DEGREES OF FREEDOM.





LOOK FOR χ^2 VALUES IN TABLES IN BACK OF BOOK.