Postclass 5 - November

3. $n = 12 \quad p = 0.45$

Using Tables:

a. $P(X \leq 5) = \boxed{0.5269}$

b. $P(X \geq 6) = 1 - P(X \leq 5)$

$= 1 - 0.5269 = \boxed{0.4731}$

c. $P(X == 7) = P(X \leq 7) - P(X \leq 6)$

$= 0.8883 - 0.7393$

$= \boxed{0.1490}$

d. $\mu = 0.45 \cdot 12 = \boxed{5.4}$

$\sigma^2 = n \cdot p \cdot (1-p) = \boxed{2.97}$

$\sigma = \sqrt{\sigma^2} = \sqrt{2.97}$

9. a. $P(X<5) =$ $P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= \binom{60}{0} \cdot .95^{60} + \binom{60}{1} \cdot .95^{59} \cdot .05 + \binom{60}{2} \cdot .95^{58} \cdot .05^2$

$+ \binom{60}{3} \cdot .95^{57} \cdot .05^3 + \binom{60}{4} \cdot .95^{56} \cdot .05^4 \cancel{+ \binom{60}{5} \cdot .95^{55} \cdot .05^5}$

$= 0.819$

b. $\lambda = n \cdot p = 3$

$$\cancel{\frac{\lambda^0 \cdot e^{-\lambda}}{0!} + \frac{\lambda^1 \cdot e^{-\lambda}}{1!}} + \frac{\lambda^2 e^{-\lambda}}{2!} + \frac{\lambda^3 e^{-\lambda}}{3!} + \frac{\lambda^4 e^{-\lambda}}{4!}$$

$\underset{\lambda=3}{P(X \leq 4)} = 0.815$ → by tables

11. $\lambda = 11$

$$P(X > 10) = 1 - P(X \leq 10)$$
$$= 1 - .460$$
$$= \boxed{0.540}$$

17. a. $\mu = \frac{n}{p} = \frac{10}{.6} = \boxed{16.67}$

$$\sigma^2 = \frac{n \cdot (1-p)}{p^2} = \boxed{11.1}$$

$$\sigma = \boxed{\sqrt{11.1}}$$

$P(X=16) =$

b. $\binom{15}{9} \cdot .6^9 \cdot .4^6 \cdot 0.6 = \boxed{0.124}$

↑ 9 successes in first 15 trials

↑ Success on the last trial

Hypergeometric

2 1. a. $P(X=x) = \dfrac{\binom{6}{x} \cdot \binom{43}{6-x}}{\binom{49}{6}}$

choose x of the state's #s to match

← choose 6-X numbers that don't match the State

← ways to choose 6 numbers

b. $\mu = n \cdot p = 6 \cdot \left(\frac{6}{49}\right) = \boxed{.7347}$

$\sigma^2 = n \cdot (p)(1-p) \cdot \left(\frac{N-n}{N-1}\right)$

$= 6 \cdot \frac{6}{49} \cdot \frac{43}{49} \cdot \left(\frac{43}{48}\right) = \boxed{.5776}$

$\sigma = \sqrt{.5776}$

c. $f(0) =$ 435 461 occurences ← most likely = 0

1 = 415 542 occurences

d. Yes: 25 m · $p(6)$ = 1.79

25 m · $p(5)$ = 461

25 m $p(4)$ = 19,125

Actual:
3
3 90
22,181

Numbers are reasonable

2.4

3. $\bar{x} = 4.956$

$s^2 = 4.134$

The poisson and binomial are good matches since $\bar{x}$ is larger, but not much larger, than s^2

5. $\bar{X} = \frac{223}{45} = 4.956$

$s^2 = 4.134$

Poisson and binomial are good fits because $\bar{X}$ is slightly larger than s^2.

7. Maximum point is when $\frac{f(x+1)}{f(x)}$ cross 1, or when

$f(x+1) < f(x)$.

So we should find when

$f(x+1) = f(x)$

$$\binom{n}{x+1} \cdot p^{(x+1)} \cdot (1-p)^{n-x-1} = \binom{n}{x} p^x (1-p)^{\cancel{n-x}}$$

$$\frac{\cancel{n!}}{(x+1)!\,(n-x-1)!} \cdot p \cdot \cancel{p^x} \cdot \frac{\cancel{(1-p)^{n-x}}}{1-p} = \frac{\cancel{n!}}{x!\,(n-x)!} \cdot \cancel{p^{(x)}} \cdot \cancel{(1-p)^{n-x}}$$

$$\frac{n-x}{(x+1)\cdot \cancel{x!} \cdot \cancel{(n-x)!}} \cdot \frac{p}{1-p} = 1 / \cancel{x(n-x)!}$$

$$n \cdot p - x \cdot p = (x+1)(1-p)$$

$$n \cdot p \cancel{-xp} = x \cancel{-xp} + 1 - p$$

$$\lceil n(p-1)+1 \rceil = x \quad \leftarrow \text{ceiling to discretize}$$

2.5

5. a. $P(X_1 = 2 \cap X_2 = 2 \cap X_3 = 5)$

$= P(X_1 = 2) \cdot P(X_2 = 2) \cdot P(X_3 = 5)$

$= \binom{4}{2} \cdot .5^2 \cdot .5^2 \cdot \binom{6}{2} \cdot \left(\frac{1}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^4 \cdot \binom{12}{5} \frac{1}{6}^5 \cdot \frac{5}{6}^7$

$= \boxed{.0035}$

b. $E(X_1 X_2 X_3) = E(X_1) \cdot E(X_2) \cdot E(X_3)$ by independence

$= 4 \cdot .5 \cdot 6 \cdot \frac{1}{3} \cdot 12 \cdot \frac{1}{6}$

$\boxed{= 8}$

c. $\text{Mean}(Y) = E(X_1 + X_2 + X_3)$ By Linearity of $E[X]$

$= E(X_1) + E(X_2) + E(X_3)$

$= 6$

$Var(Y) = Var(X_1 + X_2 + X_3)$

$= Var(X_1) + Var(X_2) + Var(X_3)$

$= 4 \cdot \frac{1}{2} \cdot \frac{1}{2} + 6 \cdot \frac{1}{3} \cdot \frac{2}{3} + 12 \cdot \frac{1}{6} \cdot \frac{5}{6}$

$= \boxed{4}$

7. Range $= \bar{X} \pm Z \cdot \frac{s}{\sqrt{n}}$

for 0.95, $Z \approx 2$ by tables

$= 11.95 \pm 2 \cdot \frac{11.8}{\sqrt{37}} = [8.07, 15.83]$

~~9. range = $\bar{x}$~~

9. Assume a binomial distribution since a letter is either successfully delivered or not

$$\bar{x} = p = \frac{142}{200} = 0.71$$

$$s^2 = \frac{p(1-p)}{n} = .0010$$

$$s = .0321$$

$$\boxed{\text{range} = \bar{x} \pm 2 \cdot s = [.6458, .7742]}$$