Probability + Statistics
November 9 – Preclass Problems

3.2

#2. $A_1 = \{x \mid 1 < x < 5\}$

So $A_1' = \{x \mid x < 1 \text{ or } x > 5\}$

Notice that A_2 is a subset of A_1', so

$$\begin{aligned} P(A_2) &\le P(A_1') \\ &\le 1 - P(A_1) \\ &\le 1 - 1/4 \\ &\le 3/4 \end{aligned}$$

#6. a) $\mu = \int_{-2}^{2} \frac{x^4 + 8x}{32}\,dx = \left[\frac{x^5/5 + 4x^2}{32}\right]_{-2}^{2} = \frac{2}{5}$

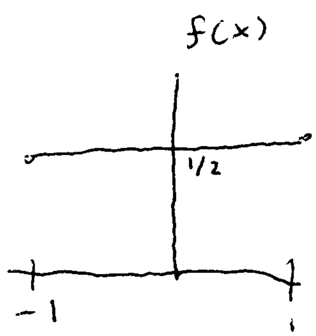
$\sigma^2 = \int_{-2}^{2} \frac{x^5 + 8x^2}{32}\,dx - \mu^2 = \frac{4}{3} - \frac{4}{25} = \frac{88}{75}$

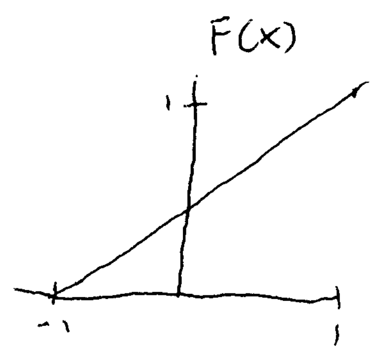
b) $\mu = \int_{-2}^{4} \frac{x^2 + 2x}{18}\,dx = \left[\frac{x^3/3 + x^2}{18}\right]_{-2}^{4} = 2$

$\sigma^2 = \int_{-2}^{4} \frac{x^3 + 2x^2}{18}\,dx - 2^2 = \left[\frac{x^4/4 + 2x^3/3}{18}\right]_{-2}^{4} - 4 = 2$

3.3

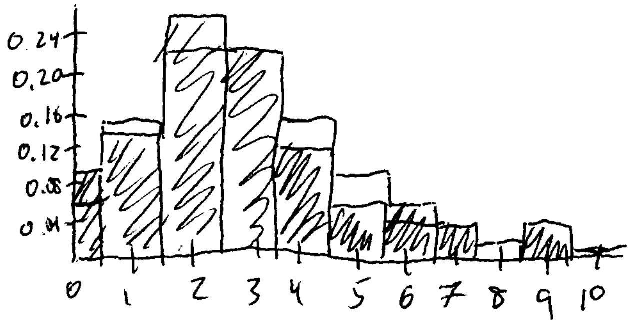
#2 $\mu = 0$, $\sigma^2 = 1/3$





#4.

a)



b) $\bar{x} = \frac{717}{35} = 20.4857$ $\quad S = 20.3437$

c) $P(X > 15) = e^{-15/20} = 0.4724$. Proportion of observations greater than 15 is $49/105 = 0.4667$

d) $P(X > 45.5 \mid X > 30.5) = e^{-45.5/20} / e^{-30.5/20} = 0.4724$

compare to 0.46.15

#6.

a) $F(x) = 1 - e^{-x/\theta}$

$\frac{1}{4} = 1 - e^{-x/\theta}$

$x = \theta \ln(4/3)$

$= 0.2877\theta$

b) $\theta - \theta \ln(4/3)$

$= \theta\,[1 + \ln(3/4)] \approx 0.7123\theta$

c) $q_3 = \theta \ln(4)$

$= 1.3863\theta$

d) $q_3 - \theta = \theta\,[\ln(4) - 1]$

$= 0.3863\theta$

$\boxed{3.4}$

#2. Use table V

a) 2.326   b) -2.576   c) 1.67   d) -2.17

#4. a) $P\left(\frac{600-650}{25} \le \frac{X-650}{25} \le \frac{660-650}{25}\right)$

$= \Phi(0.4) - \Phi(-2) = 0.6326$

b) $P\left(-\frac{c}{25} < \frac{X-650}{25} < \frac{c}{25}\right) = 0.9544$

$\therefore \frac{c}{25} = 2, \quad c = 50$

#10. Must solve $f''(x) = 0$

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$

$\ln(f(x)) = -\ln(\sigma\sqrt{2\pi}) - (x-\mu)^2/2\sigma^2$

$\frac{d}{dx} \hookrightarrow \frac{f'(x)}{fx} = -\frac{2(x-\mu)}{2\sigma^2}$

$\frac{d}{dx} \hookrightarrow \frac{f(x)f''(x) - [f'(x)]^2}{[f(x)]^2} - \frac{1}{\sigma^2}$

$f''(x) = f(x)\left[-\frac{1}{\sigma^2} + \left[\frac{f'(x)}{f(x)}\right]^2\right] = 0$

$\frac{(x-\mu)^2}{\sigma^4} = 1/\sigma^2$

$x - \mu = \pm\sigma$

$x = \mu \pm \sigma$