Your name(s):

Day 6: Continuous Probability Distributions
Sec 3.2-3.4

1. If a parachutist lands at a random point on a line between markers A and B, find the probability that he is closer to A than to B. Find the probability that his distance to A is more than three times his distance to B. Now assuming that three parachutists operate independently, what is the probability that exactly one of the three lands past the midpoint between A and B?

2. The time until failure X of an incandescent light bulb follows an exponential probability density with expected value 1000 hours.

   - Derive an expression for the CDF $F(x)$ of the RV X using the definition $F(x) = \int_{-\infty}^{x} f(t)\, dt$.
   - Calculate the probability that a light bulb will last more than 2000 hours, given that it has already lasted 1000 hours.

3. At a manufacturing plant for large diesel engines, cylinders are bored into the engine blocks and pistons are made. The machinery that makes these parts is not perfect. The inside diameter X_1 of cylinders is a normal random variable with mean 30.25 cm and standard deviation 0.06 cm. The outside diameter X_2 of pistons is normally distributed with mean 30 cm and standard deviation 0.05 cm.

   - What is the probability that a randomly chosen piston will not fit into a randomly selected cylinder? (Hint: First define a new random variable and find its distribution).
   - A given piston performs best if the clearance gap between the piston and the cylinder wall is between 0.1 and 0.35 cm. What is the probability that a randomly chosen piston performs optimally?
   - An engine has six pistons. What is the probability that an engine will be made that has at least four pistons that perform optimally?

4. The percentage of impurities per batch in a chemical product is a random variable X with PDF

$$f(y) = \begin{cases} 12x^2(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   A batch with more than 40% impurities cannot be sold. What is the mean percentage of impurities per batch? Given that a randomly selected batch does not contain more than the mean percentage of impurities, what is the probability that it cannot be sold?