

Probability & Statistics

p1/2

Post Class Problems 2/5/10 due 2/9/10

2.4 #3,5

2.5, #5,7,9

2.4 3) a. $\bar{x} = \frac{\sum x}{n} = 7.940$

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 = 7.731$$

b. Both the poisson and the binormal distributions are good matches. Since \bar{x} is only slightly larger than s^2 either would work as an approximation.

2.4 5) $\bar{x} = \frac{223}{45} = 4.956$

$$s^2 = 4.134$$

b. Both the poisson and the binormal distributions are good matches. Since \bar{x} is only slightly larger than s^2 either would work as an approximation.

~~2.5 5) $\sigma_1^2 = k, \sigma_2^2 = 2$
 $Y = 3X_2 = X_1$
 $\sigma_1^2 = 25$~~

2.5 5) $\left. \begin{array}{l} X_1: b(4, \frac{1}{2}) \\ X_2: b(6, \frac{1}{3}) \\ X_3: b(12, \frac{1}{6}) \end{array} \right\} \text{ independent random variables}$

$$\begin{aligned} \text{a) } P(X_1=2, X_2=2, X_3=5) &= P(X_1=2 \cap X_2=2 \cap X_3=5) \\ &= P(X_1=2) P(X_2=2) P(X_3=5) \\ &= \binom{4}{2} \cdot 5^2 \cdot 5^2 \cdot \binom{6}{2} \cdot \frac{1}{3}^2 \cdot \frac{2}{3}^4 \cdot \binom{12}{5} \cdot \frac{1}{6}^5 \cdot \frac{5}{6}^7 \\ &= .0035 \end{aligned}$$

$$\begin{aligned} \text{b) } E(X_1 X_2 X_3) &= E(X_1) E(X_2) E(X_3) \\ &= 4 \cdot \frac{1}{2} \cdot 6 \cdot \frac{1}{3} \cdot 12 \cdot \frac{1}{6} \\ &= 8 \end{aligned}$$

c) $Y = X_1 + X_2 + X_3$

$$\begin{aligned} \text{mean: } E(Y) &= E(X_1 + X_2 + X_3) \\ &= E(X_1) + E(X_2) + E(X_3) \\ &= 4 \cdot \frac{1}{2} + 6 \cdot \frac{1}{3} + 12 \cdot \frac{1}{6} \\ &= 6 \end{aligned}$$

2.5

5) c) continued

$$\begin{aligned}
 \text{Variance: } \text{Var}(Y) &= \text{Var}(X_1 + X_2 + X_3) \\
 &= \text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) \\
 &= 4 \cdot \frac{1}{2} \cdot \frac{1}{2} + 8 \cdot \frac{1}{3} \cdot \frac{2}{3} + 12 \cdot \frac{1}{6} \cdot \frac{5}{6} \\
 &= 4
 \end{aligned}$$

2.5

7) Range: $\bar{X} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $z_{\alpha/2} = 1.96$ (from table Vb. in back of book)note: $\alpha = 100\% - 95\%$ $\alpha = 5\% = .05$ $\alpha/2 = .0250$

$$11.95 \pm 1.96 \frac{1.18}{\sqrt{37}} = [8.07, 15.83]$$

2.5

9) Assume binomial distribution:

· letter is delivered (success)

· letter is not delivered (fail)

$$p = \frac{142}{200} = 0.71$$

$$s^2 = \frac{p(1-p)}{n} = 0.0016$$

$$s = 0.0321$$

$$z_{\alpha/2} = 1.96$$

$$p \pm z_{\alpha/2} s = 0.71 \pm 1.96(0.0321) = [.6458, .7742]$$