1) POISSON DIST APPLIES $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$; $x = 0, 1, 2, \ldots$

X_1 = NO. CARS ARRIVE AT ENTRANCE 1 $\lambda = 3$
X_2 = NO. CARS ARRIVE AT ENTRANCE 2 $\lambda = 4$

$P(\text{TOTAL 3 CARS ENTER}) = P(X_1=3, X_2=0) + P(X_1=2, X_2=1) + P(X_1=1, X_2=2) + P(X_1=0, X_2=3)$

BY INDEP.

$= P(X_1=3)P(X_2=0) + P(X_1=2)P(X_2=1) + P(X_1=1)P(X_2=2) + P(X_1=0)P(X_2=3)$

$= \frac{e^{-3}3^3}{3!} \times \frac{e^{-4}4^0}{\cancel{0!}} + \frac{e^{-3}3^2}{2!} \times \frac{e^{-4}4^1}{\cancel{1!}} + \frac{e^{-3}3^1}{\cancel{1!}} \times \frac{e^{-4}4^2}{2!} + \frac{e^{-3}3^0}{\cancel{0!}} \times \frac{e^{-4}4^3}{3!}$

$= e^{-7}\left(\frac{27}{6}\right) + e^{-7}(18) + e^{-7}(24) + e^{-7}\left(\frac{64}{6}\right) = .05213$

2) GEOMETRIC DIST. APPLIES $f(x) = q^{x-1}p$; $x = 1, 2, 3, \ldots$

X = NO. OF TRIAL ON WHICH 1ST ERROR OCCURS $p = .13$

ⓐ $P(X=3) = q^{3-1}p = (.87)^2(.13) = .0984$

ⓑ $P(X \geq 3) = 1 - P(X < 3) = 1 - (P(X=1) + P(X=2))$
$= 1 - (q^0 p + q^1 p) = 1 - (.13 + (.87)(.13))$
$= .7569$

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3) EXP. DIST. APPLIES $f(x) = \begin{cases} 1/1500 \, e^{-x/1500} & x \geq 0 \\ 0 & \text{o.w.} \end{cases}$

$\mu = 1500$ X = LENGTH OF BULB LIFE

(a) $P(X > 1000) = 1 - P(X \leq 1000) = 1 - \int_0^{1000} \frac{1}{1500} e^{-x/1500} \, dx$

$= 1 - \left(-e^{-x/1500} \Big|_0^{1000} \right) = 1 + e^{-1000/1500} - 1 = e^{-\frac{1000}{1500}}$

$= .5134$

(b) FIND m SUCH THAT $\int_0^m 1/1500 \, e^{-x/1500} \, dx = .5$
(MEDIAN)

$-e^{-x/1500} \Big|_0^m = .5 \Rightarrow -e^{-m/1500} + 1 = .5$

$\Rightarrow e^{-m/1500} = .5 \Rightarrow -\frac{m}{1500} = \ln(.5)$ $m = -1500 \ln(.5)$
$= 1039.72$

4) $X_i \sim$ POISSON $f(x_i) = \frac{e^{-\lambda} \lambda^{x_i}}{x_i !}$ WRITE LOG-LIKELIHOOD FUNCTION

$L(\lambda) = \prod_{i=1}^n f(x_i) = \prod_{i=1}^n \frac{e^{-\lambda} \lambda^{x_i}}{x_i !} = \frac{(e^{-\lambda})^n (\lambda)^{x_1 + x_2 + \cdots + x_n}}{\left(\prod_{i=1}^n x_i ! \right)}$

$\ell(\lambda) = \ln(L(\lambda)) = -n\lambda + \left(\sum_{i=1}^n x_i \right) \ln \lambda - \ln\left(\prod_{i=1}^n x_i ! \right)$

$\frac{d\ell}{d\lambda} = -n + \frac{\left(\sum_{i=1}^n x_i \right)}{\lambda} = 0 \Rightarrow \lambda = \frac{\left(\sum_{i=1}^n x_i \right)}{n} = \frac{n\bar{x}}{n} = \bar{x}$

$\frac{d^2\ell}{d\lambda^2} = (\sum x_i)(-\lambda^{-2}) = -\frac{n\bar{x}}{\lambda^2} \Big|_{\text{EVAL @ } \lambda = \bar{x}}$

$= -\frac{n\bar{x}}{\bar{x}^2} = -\frac{n}{\bar{x}} < 0$ SO IT IS A MAX!

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5) a) $P(X<.25) = \int_0^{.25} 30y^2(1-y)^2\,dy = \int_0^{.25} 30y^2(1-2y+y^2)\,dy$

$= 30\int_0^{.25}(y^2-2y^3+y^4)\,dy = 30\left(\frac{y^3}{3}-\frac{y^4}{2}+\frac{y^5}{5}\right)\Big|_0^{.25}$

$= 30y^3\left(\frac{1}{3}-\frac{y}{2}+\frac{y^2}{5}\right) = 30(.25)^3\left(\frac{1}{3}-\frac{1}{8}+\frac{1}{80}\right) = .1035$

b) USE BINOMIAL DIST $p = .1035$

X = NO. PEOPLE W/ CONGESTIVE HEART FAILURE

$P(X<2) = P(X=0) + P(X=1)$

$= \cancel{\binom{10}{0}(.1035)^0(.8965)^{10}} + \binom{10}{1}(.1035)^1(.8965)^9$

$= (.8965)^9(.8965 + .1035\times 10) = .7225$