PRINT your name: ____________________

Probability & Statistics Challenge Examination, Spring 2010

The rules for this exam are:

- Give yourself no more than 3 hours to do the exam. You may take a "free" break of up to 30 minutes (which does not count against your work time) to shower, eat, or think, but you may not do any written work while on your "free" break.

  Initial Opening of Test (Time and Date):
  Start of Break (Time and Date):
  End of Break (Time and Date):
  End of Test (Time and Date):

- The exam is closed book.
- You may use your calculator/computer to do the arithmetical calculations. Do not use your calculator/computer to do calculations other than basic arithmetic.
- Use the probability tables provided to you in PDF format.
- Please write your answers in a neat and well-organized fashion. Justify all of your answers to receive credit. Please write your answers in the boxes, when boxes are provided for your answers.
- Turn in your exam on Friday January 29th by 10 am to Mr. Paul Coveney whose office is on the second floor of Milas Hall.
- You must obtain at least a B to successfully challenge the course. The examination is thorough, covering essential elements of discrete and continuous probability, fundamental probability distributions, and essential statistics including point and interval estimation, large and small sample estimation and inference, and hypothesis testing for means and proportions.

Problem	Score
1 (10)	
2 (10)	
3 (10)	
4 (10)	
5 (15)	
6 (15)	
7 (10)	
8 (10)	
9 (10)	
10 (10)	
11 (15)	
12 (15)	
Total (140):	

1. (10) A hand of five cards is dealt at random from a standard deck. Given that the hand has three red cards and two black cards, what is the probability that the hand has three hearts ($\heartsuit$) and two clubs ($\clubsuit$)?

2. (10) A doctor is called to see a sick child. The doctor knows (prior to the visit) that 80% of the sick children in that neighborhood are sick with the flu (F) while 20% are sick with the measles (M). For simplicity, assume that M and F are complementary events. A well-known symptom of measles is a rash, denoted by R. The probability of having a rash for a child sick with the measles is 0.9. However, occasionally children with the flu also develop a rash, with probability 0.05. Upon examining the child, the doctor finds a rash. What is the probability that the child has the flu?

   Probability $= \boxed{}$

   Show your work:

3. (10) Consider the following gambling game, called the Petersburg Game: Toss a fair coin repeatedly until the first tail is observed. If the tail occurs on the first toss, you do not win anything; if the first tail occurs on the second toss, you win 2 dollars; if the first tail occurs on the third toss, you win 4 dollars. Thus, if the first tail occurs on toss $k + 1$, you win 2^k dollars, $k \geq 1$. Compute the average duration of the game. Now compute the expected winnings of the player. Are you surprised by your results?

4. (10) A parking lot has two entrances. Cars arrive at entrance I on average of 3 per hour, while cars arrive at entrance II on average of 4 per hour. If the number of cars arriving at the two entrances are independent, what is the probability that a total of three cars will enter the parking lot in a given hour?

5. (15) Consider a random variable X with the following probability distribution

$$f(x) = \begin{cases} \frac{b}{x^3} & x \geq b \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of b that makes $f(x)$ a probability density function.

(b) Determine the CDF (denoted $F(x)$) for X, and sketch its graph.

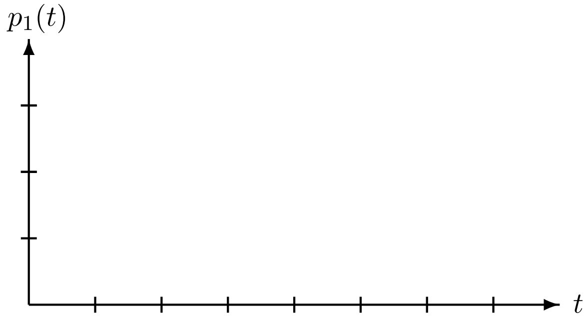
$$F(x) = \boxed{},$$

(c) Determine, if possible, the mean and variance for X.

$$E(X) = \boxed{}, \quad V(X) = \boxed{}$$

6. (15) The service life T_1 of a certain electrical device follows an exponential distribution with mean $\lambda = 10$ units of time.

(a) In the space below, record the PDF and the CDF for T_1, and sketch a qualitatively accurate graph of the PDF. Lastly, determine the probability P that the device functions for at least 10 units of time.

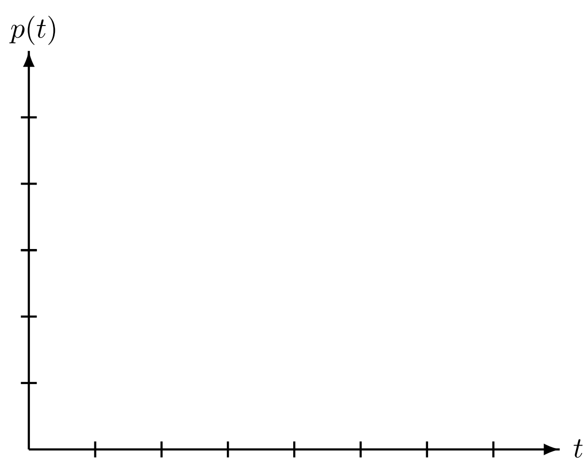


PDF $=$

CDF $=$

$P =$

(b) The electrical devices described in the previous part of this problem are shipped in lots of 100. Let $\bar{T}$ represent the average service life of the devices in a lot[1]. Describe, as accurately as you can and with a discussion of your assumptions, the probability density $p(t)$ for $\bar{T}$. Make a reasonably accurate qualitative sketch the of density; be sure to give some indication as to the scale of the axes of your graph.



[1]Thus, if $T_1, \ldots, T_{100}$ are the service lives of the 100 parts in a randomly chosen lot, then $\bar{T} = \frac{1}{n}(T_1 + T_2 + \cdots + T_{100})$ This average will vary randomly between lots—and that is the quantity of interest

7. (10) The duration of Alzheimer's disease ranges from 3 to 20 years. The average is 8 years and the standard deviation is 4 years. For a clinical study 30 patients are randomly selected that have been determined to be at the very beginning stage of the disease.

   (a) What is the probability that the average duration of the disease among the sampled patients will be less than 7 years? Discuss any assumptions that you have made.

   Probability = $\boxed{}$

   (b) What is the probability that the average duration of the disease among the sampled patients will be between 7 and 9 years?

   Probability = $\boxed{}$

8. (10) The following data represent the tensile strength of $n = 20$ samples of a certain rubber material (in units of MPa)

Tensile Strength (MPa)				
11.66	6.94	9.74	7.76	8.31
11.97	13.35	11.47	9.29	6.84
7.84	6.89	8.07	12.21	7.66
10.01	12.47	11.18	10.52	10.28

Determine a 95% two-sided confidence interval for the mean μ of the population from which these samples were drawn. Previous testing has established that the tensile strength is normally distributed in this population, and has established that the standard deviation is 2 MPa. Also determine a 99% upper confidence bound for the mean.

9. (10) A poll of 975 likely voters was taken, using a random sampling process. Of those polled, 582 indicated that they approved of a reduction in the federal income tax. Construct a 95% confidence interval for the true proportion p of voters who approve of reducing the income tax.

$$\boxed{} < p < \boxed{}$$

Show your work:

10. (10) A random sample of $n = 12$ observations from a normal population produced the following estimates: $\bar{x} = 47.1$ and $s^2 = 4.7$.

(a) Test the hypothesis $H_0 : \mu = 48$ versus $H_a : \mu \neq 48$ with $\alpha = 0.1$.

(b) What is the p-value for your test? p-value = $\boxed{}$

(c) Find a 90% confidence interval for the mean, and interpret this interval.

$$\boxed{} < \mu < \boxed{}$$

11. (15) A political poll during the 2000 presidential election asked a random sample of 500 male and 500 female voters which candidate they preferred. The data are summarized in the following table:

GROUP	n	Prefer Bush	Prefer Gore
Male	500	337	163
Female	500	215	285

Test the hypothesis that the proportions of males and females prefering candidate Bush are the same. Fill in the blanks, and show your scratch work on this page.

(a) List the assumptions that you made to perform the test and assess their plausibility.

(b) Compute the relevant test statistic and its p-value for the two-sided test of the hypothesis that the difference of proportions is zero, against the alternative that the proportions are different:

Test statistic = $\boxed{}$, p-value = $\boxed{}$

(c) Find a 95% confidence interval for the difference in proportions, and interpret this interval.

$\boxed{} < p_{\text{male}} - p_{\text{female}} < \boxed{}$

(d) Do you reject or fail to reject the hypothesis at the 95% significance level?

12. (15) A manufacturer of concrete claims that his product has a reasonably stable compressive strength, measured in units of kilograms per square centimeter. He reports that the range of strength observed over many batches of concrete is 40 kg/cm^2. Suppose that you are considering purchasing concrete from this manufacturer, but you must be convinced that the variability in strength is no greater than what is claimed. Your engineers take a random sample of $n = 10$ batches of the material and test their strength, giving sample estimates of $\bar{x} = 312$ and $s^2 = 195$. If you assume that the range of a normal distribution is about 4σ, test the hypothesis that the variance in strength of the sampled concrete is consistent with the manufacturer's claims. Use $\alpha = 0.05$ and a one-sided test.

(a) What is the appropriate test statistic to use, with how many degrees of freedom?

Test statistic = $\boxed{}$, df = $\boxed{}$

(b) Your null hypothesis is H_0 : $\boxed{}$ versus H_a : $\boxed{}$

(c) What is the value of your test statistic = $\boxed{}$

(d) Critical value of test statistic ($\alpha = 0.05$) = $\boxed{}$

(e) Find a 95% confidence interval for the variance, and interpret this interval.

$$\boxed{} < \sigma^2 < \boxed{}$$

(f) Do you reject the hypothesis, or fail to reject it?