In-Class Problems, Day 2

26 October 2009

1 a. Permutation of 68 students

$${}_{68}P_{68} = \boxed{68!}$$

b $\boxed{P(X) = \frac{1}{68}}$

c. $P(X') = 1 - P(X) = \boxed{\frac{67}{68}}$

d. $P(Y) = \boxed{\frac{1}{68}}$

e. Since X and Y are mutually exclusive:

$P(X \cap Y) = 0$ ~~Ex 1.1-6~~ - p3

$P(X \cup Y) = \frac{2}{68}$ - Defn 1.1-1, p 6

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = 0$ - Defn 1.3-1, p19

2. Choose 4 students from 300

$${}_{300}C_4 = \frac{300!}{4!\,296!} = \boxed{\frac{300 \cdot 299 \cdot 298 \cdot 297}{4 \cdot 3 \cdot 2 \cdot 1}}$$ Defn 1.2-6

3. Ball 1 has a choice between 365 boxes
Ball 2 " " 364 boxes
3 363
⋮ ⋮
Ball 40 326 boxes

By multiplication principle, number of outcomes equal $365 \cdot 364 \cdot 321 \cdot \ldots \cdot 326$

$$= {}_{365}P_{40} = \frac{365!}{325!}$$ Also Defn 1.2-5, p13

4. To fill P, we can choose between 300 students

To fill VP, " " 299 → every student but the president

" " T, " " 298

" " C, " " 297

By multiplication principle, total ways equal

$$300 \cdot 299 \cdot 298 \cdot 297 = {}_{300}P_4 = \frac{300!}{296!}$$

5. choose 5 cards from 52

$$\boxed{{}_{52}C_5 = \frac{52!}{5!\,47!}}$$ 1.2-6, p 14

Hands with 4 diamonds equal

hands with exactly 4 diamonds

+ hands with 5 diamonds

4 Diamonds: $\underset{\substack{\uparrow \\ 13 \text{ diamonds}}}{{}_{13}}\,C\,\underset{\substack{\uparrow \\ \text{chose } 4}}{4} \cdot \underset{\substack{\uparrow \\ 39 \text{ non-diamond}}}{{}_{39}}C_1$

$$\boxed{\text{Total} = {}_{13}C_4 \cdot 39 + {}_{13}C_5}$$

5 Diamonds: ${}_{13}C_5$

$$P(\text{4-diamonds}) = \frac{\#\text{ of hands w/ 4 diamonds}}{\#\text{ of 5-card hands}}$$

$$= \cancel{{}_{52}C_5} \quad \frac{{}_{13}C_4 \cdot 39 + {}_{13}C_5}{{}_{52}C_5}$$

$$= \frac{29{,}172}{2{,}598{,}960}$$

6. $P(\# = 2 \mid \# = \text{even})$

Define $X := \text{Die} == 2$

Define $Y := \text{Die} == \text{even}$

$$P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$$ Defn. 1.3-1 p 19.

$P(X \cap Y) = \frac{1}{6}$ because $P(X) = 1/6$ and X is a subset of Y

$P(Y) = \frac{1}{2}$

$$\boxed{P(X \mid Y) = \frac{1}{3}}$$

There are 2^{10} possible flip combinations

7. To get the 5^{th} head on the 10^{th} flip, we need to flip exactly 4 heads and 5 tails ~~and~~ on the first 9 flips, then flip a head on the last one.

$$\overbrace{F\ F\ F\ F\ F\ F\ F\ F\ F}^{\text{choose 4 flips to be heads}}$$

~~The~~

4 heads and 5 tails means we choose 4 of the first 9 flips to be heads. There are ${}_9C_4$ ways to do so.

There's 1 way to flip a head on the last flip

$$P(X) = \frac{{}_9C_4}{2^{10}} = \boxed{\frac{126}{1024}}$$

8. $P(X) :=$ Probability exactly 4 out of first 14 couples agree

$P(Y) :=$ Probability 15^{th} couple agree $= 0.2$

~~By~~ ~~Mu~~ $P(X \cap Y) = P(X) \cdot P(Y) \rightarrow$ Probability the 15^{th} couple she asks is the 5^{th} to participate

$$P(X) = \underbrace{\underset{\substack{\uparrow \\ \text{14 couples}}}{{}_{14}}\underset{\substack{\uparrow \\ \text{choose} \\ \text{4 to agree}}}{C_4} \cdot 0.2^4 \cdot 0.8^{10}}_{\substack{\text{binomial} \\ \text{distribution}}} = 0.172$$

$P(Y) = 0.2$

$$P(X \cap Y) = 0.2 \cdot 0.172 = \boxed{0.0344}$$