1) TAKE RS OF SIZE n; $X_1,\dots,X_n \ni X_i \sim f(x_i) = \lambda e^{-\lambda x_i}$ $x_i \geq 0$

WRITE LIKELIHOOD FUNC

$$L(\lambda) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} = (\lambda)^n e^{-\lambda\left(\sum_{i=1}^{n} x_i\right)} = \lambda^n e^{-\lambda n \bar{x}}$$

TAKE $\ln(\cdot) \Rightarrow \ell(\lambda) = n\ln(\lambda) + -\lambda n\bar{x}\ln e = n\ln(\lambda) - n\bar{x}\lambda$

NOW

$$\frac{d}{d\lambda}\ln(\cdot) = \frac{n}{\lambda} - n\bar{x} = 0 \Rightarrow \frac{n}{\lambda} = n\bar{x} \Rightarrow \hat{\lambda} = \frac{1}{\bar{X}} \text{ IS C.P.}$$

$$\frac{d^2}{d\lambda^2}\ln(\cdot) = \frac{-n}{\lambda^2} < 0 \text{ ALWAYS } \therefore \boxed{\hat{\lambda} = \frac{1}{\bar{X}} \text{ IS M.L.E.}}$$

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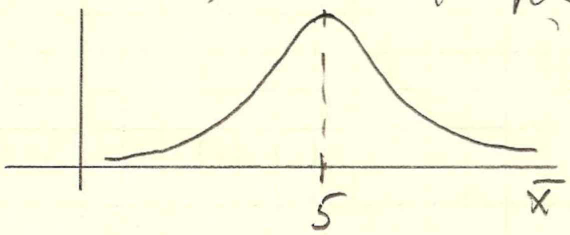
2) $X \sim$ UNIFORM $\quad \mu = \int_4^6 x\frac{1}{2}\,dx = \frac{1}{2}\frac{x^2}{2}\Big|_4^6 = \frac{36-16}{4} = 5$

$$\sigma^2 = \int_4^6 x^2\frac{1}{2}\,dx - (5)^2 = \frac{x^3}{6}\Big|_4^6 - 25 = \left(36 - \frac{64}{6}\right) - 25$$

$$= 36 - \frac{32}{3} - 25 = \frac{108-32-75}{3} = \frac{1}{3}$$

SO BY CLT, $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) = N\left(5, \frac{1}{10\sqrt{3}}\right)$



$\mu_{\bar{x}} = 5$

$\sigma_{\bar{x}} = \frac{1}{10\sqrt{3}}$

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3) $n = 49$ WANT 60% CI (TWO-SIDED) USE t-DIST. SINCE WE DON'T KNOW σ

(a) $\bar{x} = 127 \Rightarrow 100(1-\alpha)\% = 60\% \Rightarrow \alpha = .4$

$s = 52$ CI $\Rightarrow \bar{x} \pm t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}} \simeq \bar{x} \pm z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}$ (SINCE $n>30$) t-TABLES ONLY GO UP TO $n=31$

$z_{.2} \simeq .845$

$$127 \pm (.845)\frac{52}{\sqrt{49}} = \begin{cases} 133.277 \\ 120.723 \end{cases}$$

$\boxed{(120.723, 133.277)}$

(b) 60% OF CIs GENERATED IN THIS MANNER SHOULD CONTAIN μ

4) $n = 985$ $\Rightarrow \hat{p} = \frac{Y}{n} = \frac{592}{985}$ WANT 95% CI $\Rightarrow \alpha = .05$

Y = NO. APPROVE $= 592$ $\hat{q} = \frac{393}{985}$ TWO-SIDED

$\bar{X} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \simeq \hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$z_{.025} = 1.96$

$\hat{p} = .601$

$\hat{q} = .399$

$\Rightarrow .601 \pm (1.96)\sqrt{\frac{(.601)(.399)}{985}}$ $.6316$ / $.5704$

SO CI IS $(.5704, .6316)$

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5) ⓐ USE CI BASED ON χ^2 STAT $= (n-1)S^2/\sigma^2$ FOR σ^2

$\left(\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}}, \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}}\right)$ w/ $n-1$ DOF

THEN TAKE $\sqrt{\ }$ OF BOTH SIDES

$\Downarrow$

$\alpha = .05 \Rightarrow \chi^2_{.025} = 27.49$

$n = 16$

15 DOF

$\chi^2_{.975} = 6.262$

$\left(\sqrt{\frac{(15)(2.43)^2}{27.49}}, \sqrt{\frac{(15)(2.43)^2}{6.262}}\right)$

$(.3424, 1.503)$

ⓑ USING EXPRESSIONS FROM ⓐ ONLY w/ $S^2 = 2.39$

$n = 13$ (12 DOF)

$\chi^2_{.025} = 24.74$

$\chi^2_{.975} = 5.009$

$\left(\sqrt{\frac{(13)(2.39)^2}{24.74}}, \sqrt{\frac{(13)(2.39)^2}{5.009}}\right)$

$(.3483, 1.720)$

ⓒ CI FOR RATIO OF VARS σ_X^2/σ_Y^2 BASED ON $F = \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2}$ IS

CORRECT F-STAT

$\left(\frac{S_X^2/S_Y^2}{F_{\frac{\alpha}{2};\, n-1, m-1}}, F_{\frac{\alpha}{2};\, m-1, n-1} \frac{S_X^2}{S_Y^2}\right)$

$\alpha = .05$

$n = 16$

$m = 13$

$\Rightarrow F_{.025;\, 15, 12} = 3.18$

$F_{.025;\, 12, 15} = 2.96$

$\Downarrow$

$\left(\frac{(2.43)^2}{(2.39)^2(3.18)}, \frac{(2.43)^2(2.96)}{(2.39)^2}\right) = \boxed{(.3251, 3.06)}$

ⓓ NO EVIDENCE TO SUPPORT DIFF. IN POP. VARIANCES SINCE CI'S FOR σ_X & σ_Y FROM ⓐ AND ⓑ OVERLAP.

ALSO, IF $\sigma_X^2 = \sigma_Y^2 \Rightarrow \frac{\sigma_X^2}{\sigma_Y^2} = 1 \in$ CI $(.3251, 3.06)$ FROM ⓒ

$\therefore$ NO EVIDENCE OF A DIFFERENCE.

5 (c) $F = \dfrac{S_X^2/\sigma_Y^2}{S_Y^2/\sigma_X^2}$ INSTEAD OF $F = \dfrac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2}$

RESULTING CI EXPRESSION IS

$$\left(\frac{S_Y^2/S_X^2}{F_{\frac{\alpha}{2}, n-1, m-1}}, F_{\frac{\alpha}{2}; m-1, n-1}\frac{S_Y^2}{S_X^2}\right) = \left(\frac{(2.39)^2}{(2.43)^2(3.18)}, \frac{(2.39)^2(2.96)}{(2.43)^2}\right)$$

$$= \boxed{(.3042, 2.863)}$$

(d) SAME CONCLUSION: SINCE $1 \in$ CI $(.3042, 2.863)$
THERE IS NO EVIDENCE TO SUGGEST A DIFFERENCE