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Probability and Statistics, Fall 2009, Quiz 1

Sections 1.4-1.5, 2.1-2.2, Independence, Bayes's, Discrete Probability Distributions, Expectations

- No resources are allowed, except for a calculator/computer for basic arithmetic; do not use any pre-programmed formulas.
- Write all answers in order on additional sheets of paper. Staple all pages together.
- Clearly explain your answers and show your work.
- There is a strict 90 minute limit for the quiz. Set an alarm. Aim for 60 minutes.

  Initial Quiz Download/View (Time and Date):
  End of Quiz (Time and Date):

1. let X be the value you get when tossing a single fair die.
   (a) write down the probability mass function for X.
   (b) compute the mean of X, that is $E(X)$.
   (c) compute $E(X^2)$.
   (d) compute the variance of X.

2. A test can detect organic pollutants (OP) with 99.7% accuracy, volatile solvents (VS) with 99.95% accuracy, and chlorinated compounds (CC) with 89.7% accuracy. A test signals positive if any of these three types of pollutants are found, and it signals negative if none of these pollutants is found. Test samples taken for calibration showed 60% of samples are contaminated with OP, 27% are contaminated with VS, and 13% are contaminated with CC. A test sample is selected randomly. What is the probability that it will signal positive? If the signal is positive, what is the probability that the sample is contaminated with CC?

3. The ability to observe and recall details is important in science and in situations like legal trials. Unfortunately, the power of suggestion can distort memory. The following is motivated by a 1978 study by Loftus et al (*Journal of Experimental Psychology: Human Learning and Memory*, 4, p. 19-31). Subjects are shown a film in which a car is moving along a country road. There is no barn in the film. The subjects are then asked a series of questions concerning the film. Half of the subject are asked, "How fast was the car moving when it passed the barn?" The other half is not asked this question. Later, each subject is asked, "Is there a barn in the film?" Of those asked the first question concerning the barn, 17% answer "yes;" only 3% of the others answer "yes."
   (a) What is the probability that a randomly selected subject in this study claims to have seen the nonexistent barn?
   (b) Is claiming to see the barn independent of being asked the first question about the barn? (Justify mathematically.)

4. During a particular period, the IT department received 20 service orders for problems with printers, of which 8 were laser printers and 12 were inkjet printers. A sample of 5 of these service orders is to be selected for inclusion in a customer satisfaction survey. Suppose that the 5 are selected randomly, so that any particular subset of size 5 has the same chance of being selected as does any other subset. Let X be the random variable counting the number of inkjet problems selected in the sample of 5.

   (a) What is the probability that $X = 2$?

   (b) What is the probability mass function for X?

5. The National Hockey League playoffs have just started, and opening day at Fenway park heralds the beginning of the baseball season. What do these two sports have in common? They share (along with the NBA) the same format for their final championship playoff series—the first team to win four games in the final series wins the title, and no tie games are possible. Thus a championship series must last at least four games (the dreaded "sweep" for the losing team) and can last at most seven games (much loved by advertisers and league promoters). Suppose in a given championship there are two teams, which we call teams A and N. Suppose that the probability of team A defeating team N in any given game is constant and equal to p, with $0 < p < 1$, and suppose that the outcome of any game is independent of the others. Thus, if $p > \frac{1}{2}$, team A is favored over team N. If $p = \frac{1}{2}$ the contest is "fair."

   Let the random variable X be the number of games played in such a series to determine the winner. Thus X takes the (random) values 4, 5, 6, or 7. Do your calculations on the scratch page provided, and write your results in the space provided on this page.

   (a) Determine the PMF for X.

   (b) Determine an expression for the expected duration of the series, $E[X]$.