QUIZ #3 SOLS

1) $L(\lambda) = \prod_{i=1}^{n} f(x_i)$

$= \prod_{i=1}^{n} \lambda e^{-\lambda x_i} = \lambda^n e^{-\lambda(\sum x_i)} = \lambda^n e^{-\lambda n\bar{x}}$

$n\bar{x} = \frac{n}{\lambda}$

$\lambda = \frac{1}{\bar{x}}$

$l(\lambda) = \ln(L(\lambda)) = n\ln\lambda - \lambda n\bar{x}$ $\quad \frac{dl}{d\lambda} = \frac{n}{\lambda} - n\bar{x} = 0$

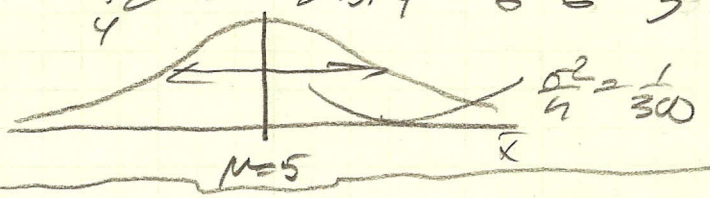
$\frac{d^2l}{d\lambda^2} = -\frac{n}{\lambda^2} < 0$ ALWAYS. THUS $\boxed{\lambda = \frac{1}{\bar{x}} \text{ IS M.L.E.}}$

2) $\mu = \int_4^6 \frac{1}{2} x\,dx = \frac{1}{2}\frac{x^2}{2}\Big|_4^6 = \frac{36}{4} - \frac{16}{4} = \frac{20}{4} = 5$ $\quad \sigma^2 = \frac{76}{3} - \frac{75}{3} = \frac{1}{3}$ $\quad n = 100$

$E(X^2) = \int_4^6 \frac{1}{2} x^2\,dx = \frac{1}{2}\frac{x^3}{3}\Big|_4^6 = \frac{216}{6} - \frac{64}{6} = \frac{76}{3}$

SO $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

$\boxed{\mu = 5 \quad \frac{\sigma^2}{n} = \frac{1}{300}}$



3) $n = 49$ $\quad \bar{x} = 127$ $\quad s = 52$ $\quad \alpha = .4$ $\quad z_{.2} = .845$

TYPICALLY USE t-DIST, BUT IS LARGE ENOUGH TO USE Z

a) $\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \rightarrow 127 \pm (.845)\frac{(52)}{\sqrt{49}}$ $\quad$ 133.27, 120.72

$\boxed{(120.72, 133.27)}$

b) $\boxed{\text{WOULD EXPECT 60\% OF THEM TO CONTAIN } \mu}$

4) $n = 800$ $\quad \hat{p} = .45$ $\quad \hat{q} = .55$ $\quad \alpha = .02$ $\quad z_{.01} = 2.325$

$\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} \rightarrow .45 \pm (2.325)\sqrt{\frac{(.45)(.55)}{800}}$ $\quad$ .4908, .4091

$\boxed{(.4091, .4908)}$

SINCE $\mu = .5 \notin$ CI CAN'T CONCLUDE MAJORITY OF ADULTS THINK STORES BETTER

5) a) USE $\chi^2 = (n-1)\frac{S^2}{\sigma^2}$ CI FOR σ IS $\left(\sqrt{\frac{(n-1)}{\chi^2_{\frac{\alpha}{2}}}}S, \sqrt{\frac{(n-1)}{\chi^2_{1-\frac{\alpha}{2}}}}S\right)$

$n=16$ $\chi^2_{.025} = 27.49$
$\alpha = .05$
15 DOF $\chi^2_{.975} = 6.262$

$\left(\sqrt{\frac{15}{27.49}}(2.43), \sqrt{\frac{15}{6.262}}(2.43)\right)$

CI IS $(1.795, 3.76)$

b) $n=13$ $\chi^2_{.025} = 23.34$
$\alpha = .05$
12 DOF $\chi^2_{.975} = 4.404$

$\left(\sqrt{\frac{12}{23.34}}(2.39), \sqrt{\frac{12}{4.404}}(2.39)\right) = (1.713, 3.945)$

c) FOR STATISTIC $F = \frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2}\frac{(m-1)}{(n-1)}$

CI IS $\left(\frac{1}{F_{\frac{\alpha}{2}}(n-1, m-1)}\frac{S_X^2}{S_Y^2}, F_{\frac{\alpha}{2}}(m-1, n-1)\frac{S_X^2}{S_Y^2}\right)$ $\alpha = .05$

$= \left(\frac{1}{3.18}\frac{(2.43)^2}{(2.39)^2}, 2.96\frac{(2.43)^2}{(2.39)^2}\right)$

$F_{.025}(12,15) = 2.96$
$F_{.025}(15,12) = 3.18$
$m=13$ $n=16$

$= (.325, 3.06)$

d) CIs FROM PARTS (a) AND (b) OVERLAP
CI FROM PART (c) CONTAINS $\frac{\sigma_X^2}{\sigma_Y^2} = 1$ (SAME VARS)

∴ NO SIG. DIFFERENCE IN VARIABILITY BETW. MALES AND FEMALES!