Probability & Statistics

Preclass Problems: 1/26/10
Sec 1.3 #2,4,6
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.3 2) Select one person randomly

a) $P(A_1) = \frac{1041}{1456} = \frac{\text{People in } A_1 \text{ Category}}{\text{Total \# People}}$

b) $P(A_1|B_1) = \frac{P(A_1 \cap B_1)}{P(B_1)}$

$= \frac{392/1456}{633/1456}$

$= \frac{392}{633}$

c) $P(A_1|B_2) = \frac{649/1456}{823/1456}$

$= \frac{649}{823}$

d) The probability that a person will favor a gun law (A_1) given that they are a ~~wo~~man (B_1) is $392/633 = 61.9\%$ whereas the probability that a woman will favor a gun law is $649/823 = 78.9$
The proportion of women ~~th~~ who favor a gun law is greater than the proportion of men.

1.3 4) a) $P(\heartsuit, \heartsuit) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$ note: no replacement

b) $P(\heartsuit, \clubsuit) = \frac{13}{52} \cdot \frac{13}{51} = \frac{13}{204}$

c) $P(\heartsuit, A) = P(\text{non}A\ \heartsuit, A) + P(A \text{ of } \heartsuit, A)$

note: to solve this problem you sum up the mutally exclusive ev
- getting a heart that is not an ace; then getting an ace
- getting the ace of hearts; then getting an ace

$= \frac{12}{52} \cdot \frac{4}{51} + \frac{1}{52} \cdot \frac{3}{51} = \frac{51}{52 \cdot 51} = \frac{1}{52}$

1.3 6) a) $P(\text{like both}) = \frac{8}{14} \cdot \frac{7}{13} = \frac{4}{13}$

b) $P(\text{like niether}) = \frac{6}{14} \cdot \frac{5}{13} = \frac{15}{91}$

c) $P(\text{like exactly } 1) = P(\text{like only } 1^{st}) + P(\text{like only } 2^{nd})$

$= \frac{8}{14} \cdot \frac{6}{13} + \frac{6}{14} \cdot \frac{8}{13}$

$= 2\left(\frac{8}{14} \cdot \frac{6}{13}\right) = \frac{48}{91}$

OR $= P(\text{like both, or like niether})'$ note: compliment

$= 1 - P(\text{like both or like niether})$

$= 1 - \left[\frac{4}{13} + \frac{15}{91}\right]$

$= 48/91$

1.4 2) a) $P(A \cup B)$ when A and B are independent

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - (0.3)(0.6)$

$= 0.72$

b) $P(A|B)$ when A and B are mutually exclusive

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0$

note: when events are mutually exclusive the probability they will occur together is $\emptyset$.

1.4 4) A, B, C are independent meaning $P(A \cap B \cap C) = P(A)P(B)P(C)$

Show the following are independent:

a) $P(A \cap (B \cap C)) = P(A \cap B \cap C)$ by definition of intersection

$= P(A)P(B)P(C)$ by independence

$= P(A)P(B \cap C)$

b) $P(A \cap (B \cup C)) = P((A \cap B) \cup (A \cap C))$ by distributive property

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$

$= P(A)(P(B) + P(C) - P(B)P(C))$

$= P(A)P(B \cup C)$

4) Continued . . .

c) $P(A' \cap (B \cap C')) = P(A' \cap B \cap C')$ definition of intersection

$= P(A' \cap C' \cap B)$ Commutative property

$= P(B)P((A' \cap C') | B)$ definition of conditional probabili

$= P(B)[1 - P((A \cup C)|B)]$ definition of compliment

$= P(B)[1 - P(A \cup C)]$ since A, B, C are independent no conditional probability applie

$= P(B)P((A \cup C)')$ definition of compliment

$= P(B)P(A' \cap C')$ deMorgan's Law

$= P(B)P(A')P(C')$ definition of independence

$= P(A')P(B)P(C')$

$= P(A')P(B \cap C')$

d) $P(A' \cap B' \cap C') = P((A \cup B \cup C)')$ deMorgan's Law

$= 1 - P(A \cup B \cup C)$ definition of compliment

$= 1 - [P(A) + P(B) + P(C) - (P(A \cap B) + P(A \cap C) + P(B \cap C) - P(A \cap B \cap$

$= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + P(B)P(C) - P(A)P(B)P(C)$

$= [1 - P(A)][1 - P(B)][1 - P(C)]$

$= P(A')P(B')P(C')$