DAY 10 - INCLASS SOLS

1) X_1 = CYLINDER DIAMETER $\sim N(\mu_1 = 30.25, \sigma_1^2 = .06^2)$

(a)

X_2 = PISTON DIAMETER $\sim N(\mu_2 = 30, \sigma_2^2 = .05^2)$

DEFINE $Y = X_1 - X_2$ (GAP) $\sim N(\mu_Y, \sigma_Y^2) \ni$ $\mu_Y = \mu_1 - \mu_2 = .25$

$\sigma_Y^2 = \sigma_1^2 + \sigma_2^2 = .0061$

NOT FIT $\Rightarrow Y < 0$

$P(Y<0) = P\left(\frac{Y-\mu}{\sigma} < \frac{0-\mu_Y}{\sigma_Y}\right) = P\left(Z < \frac{.25}{\sqrt{.0061}}\right) = P(Z < -3.21) = .0007$

(b) $P(.1 \le Y \le .35) = P\left(\frac{.1-.25}{.0061} \le \frac{Y-\mu_Y}{\sigma_Y} \le \frac{.35-.25}{.0061}\right) = P(-1.92 \le Z \le 1.28)$

$= .8997 - .0287 = .871$

(c) $p = .871$, $n = 6$, $X = 4$ USE BINOMIAL $P(X \ge 4) = P(X=4) + P(X=5) + P(X=6) =$

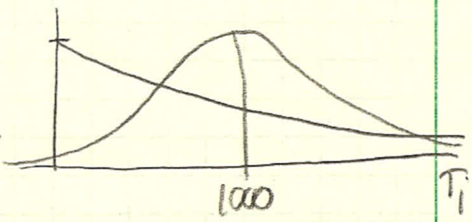
$\binom{6}{4}(.871)^4(.129)^2 + \binom{6}{5}(.871)^5(.129) + \binom{6}{6}(.871)^6(.129)^0 =$

$(.871)^4(15(.129)^2 + 6(.871)(.129) + (.871)^2) = .968$

2) $T_i \sim exp(\mu, \sigma^2)$ $\mu = 1000$ $\sigma^2 = 1000^2$

BY THE CLT $\bar{T} \sim N(\mu_{\bar{T}}, \sigma^2_{\bar{T}}) \ni \mu_{\bar{T}} = 1000$

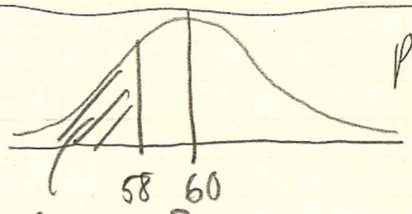
$\sigma^2_{\bar{T}} = \frac{\sigma^2}{n} = \frac{1000^2}{n}$



3) $\sigma = 10$, $n = 100$ FIND $P(|\bar{X} - \mu| < 1) = P(-1 \le \bar{X} - \mu \le 1) = P\left(\frac{-1}{\sigma/\sqrt{n}} \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le \frac{1}{\sigma/\sqrt{n}}\right)$

$= P(-1 \le Z \le 1) = .8413 - .1587 = .6826$

4) $\mu = 60$, $\sigma = 8$, $n = 100$, $\bar{X} = 58$



$P(\bar{X} < 58) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le \frac{58-60}{.8}\right) = P(Z < -2.5)$

$= .0062$ VERY LOW PROB! NOT A COINCIDENCE. THAT H.S. HAS ABNORMALLY LOW TEST SCORES!

5) $X \sim b(n,p)$ $P(X \le 8) = .274$ $P(X=8) = P(X \le 8) - P(X \le 7) = .274 - .154$ $= .120$

$n = 25$

$p = .4$ NOW USE NORMAL APPROXIMATION $\mu = np = 10$, $\sigma^2 = npq = 6$

$P(X \le 8) \doteq P(X \le 8.5) = P\left(\frac{X-\mu}{\sigma} \le \frac{8.5-10}{\sqrt{6}}\right) = P(Z \le -.61) = .2709$

$P(X = 8) \doteq P(7.5 \le X \le 8.5) = P\left(\frac{7.5-10}{\sqrt{6}} \le Z \le \frac{8.5-10}{\sqrt{6}}\right) = P(-1.02 \le Z \le -.61)$

$= .2709 - .1539 = .1587$

6) $n=100$ FIND $P(\frac{X}{n} > .55) = P\left(\frac{\frac{X}{n} - .5}{\sqrt{.0025}} \geq \frac{.55 - .5}{\sqrt{.0025}}\right) = P(Z \geq 1)$

MEAN $= p = .5$ $= .1587$

VAR $= \cancel{\frac{pq}{n}} = \frac{.5^2}{100} = .0025$

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7) POISSON w/ $\lambda = 100$ FIND $P(X \leq 110)$ USE NORMAL APPROX

$\mu = \lambda = 100$

$\sigma^2 = \lambda = 100$ $P\left(\frac{X-\mu}{\sigma} \leq \frac{110-100}{10}\right) = P(X \leq 1) = 1 - P(X > 1)$

$\sigma = 10$ $= 1 - .1587 = .8413$