Pre-class

29 Oct. 2009

1.4

2. a. $P(A \cap B) = P(A)P(B) = 0.18$ ← by independence

$P(A \cup B) = P(A) + P(B) - P(A \cup B)$

$= 0.3 + 0.6 - 0.18$

$\boxed{= 0.72}$

b. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{P(B)} = \boxed{0}$ ← by mutually exclusive

1.4 4. Independence implies $P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap (B \cap C)) = P(A \cap B \cap C)$ by defn of intersection

$= P(A)\,P(B)\,P(C)$ by independence

$= P(A)\,P(B \cap C)$

$P((B \cup C) \cap A) = P[(A \cap B) \cup (A \cap C)]$ by distributive property

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$ since A, B, C are independent

$= P(A)[P(B) + P(C) - P(B \cap C)]$

$= P(A) \cdot P(B \cup C)$

$$P[A' \cap (B \cap C')] = P(A' \cap C' \cap B) \quad \text{commutative property}$$

$$= P(B)[P(A' \cap C' \mid B)] \quad \text{Defn of conditional probability}$$

$$= P(B)[1 - P((A \cup C) \mid B)] \quad \text{Defn of complement}$$

$$= P(B) \cdot [1 - P(A \cup C)] \quad \text{since } A, B, C \text{ are independent}$$

$$= P(B)\, P[(A \cup C)'] \quad \text{Defn of complement}$$

$$= P(B)\, P(A' \cap C') \quad \text{De Morgen's Law}$$

$$= P(B)\, P(A')\, P(C') \quad \text{Defn of independence}$$

$$= P(A')\, P(B)\, P(C')$$

$$= P(A')\, P(B \cap C')$$

$$P[A' \cap B' \cap C'] = P[(A \cup B \cup C)'] \quad \text{Defn of De Morgen}$$

$$= 1 - P(A \cup B \cup C) \quad \text{Defn of complement}$$

$$= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + P(B)P(C) - P(A)P(B)P(C) \quad \uparrow \text{defn of union}$$

$$= [1 - P(A)][1 - P(B)][1 - P(C)]$$

$$= P(A')\, P(B')\, P(C')$$

1.5

2. a $P(G) = P(A)\,P(G|A) + P(B)\cdot P(G|B)$

$= 0.4 \cdot .85 + 0.6 \cdot .75$

$= 0.79$

b. $P(A|G) = \dfrac{P(A \cap G)}{P(G)}$ Bayes Theorem

$= \dfrac{0.9 \cdot 0.85}{0.79} = \boxed{0.9309}$

4. $P(16\text{-}25 \mid A) = \dfrac{P(16\text{-}25 \cap A)}{P(A)}$

A = accident

$= \dfrac{0.05 \cdot 0.10}{0.05 \cdot .10 + 0.02 \cdot .55 + 0.03 \cdot 0.2 + 0.09 \cdot 0.15}$

$= \dfrac{0.005}{0.005 + .011 + .006 + .006} = \boxed{0.179}$

6. $P(\text{standard} \mid \text{death}) = \dfrac{P(\text{standard} \cap \text{death})}{P(\text{death})}$

$= \dfrac{0.6 \cdot 0.01}{.0106 + 0.008 \cdot .3 + .007 \cdot .1} = \dfrac{.006}{.0091} = \boxed{0.659}$

$$P(\text{preferred} \mid \text{death}) = \frac{0.008 \cdot 0.3}{0.091} = \boxed{0.264}$$

$$P(\text{ultrapreferred} \mid \text{death}) = \frac{0.007 \cdot 0.1}{0.091} = \boxed{0.077}$$