DAY 5 - INCLASS SOLS

1) a) $\frac{1}{n}$

b) $\left(\frac{n-1}{n}\right)\left(\frac{1}{n-1}\right) = \frac{1}{n}$ $\qquad$ $\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n-1}\right)\left(\frac{1}{n-2}\right) = \frac{1}{n}$

c) A = GETS ACCESS ≡ 1ST TRY OR 2ND TRY OR 3RD TRY ← M.E.

$P(A) = P(\text{1ST TRY}) + P(\text{2ND TRY}) + P(\text{3RD TRY}) = \frac{3}{n} = \frac{3}{7}$

d) X = NO. TRIES $\quad x = 1, 2, \dots, 7 \quad P(X = x) = 1/7$

$\mu = E(X) = \sum_x x\,p(x) = \left(\frac{1}{7}\right)(1+2+3+4+5+6+7) = \frac{28}{7} = 4$

$E(X^2) = \sum_x x^2 p(x) = \left(\frac{1}{7}\right)(1^2+2^2+3^2+4^2+5^2+6^2+7^2) = \frac{140}{7} = 20$

$\sigma^2 = E(X^2) - \mu^2 = 20 - 16 = 4$

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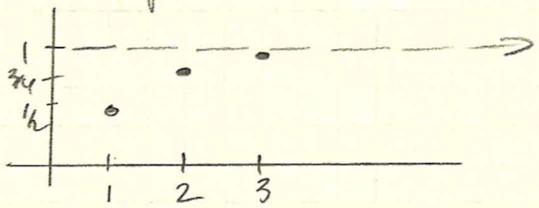
2) $F(x) = P(X \le x) = \sum_{k=1}^{x} (1-p)^{k-1} p = p + p(1-p) + p(1-p)^2 + \dots + p(1-p)^{x-1}$

$= p\underbrace{(1 + q + q^2 + \dots + q^{x-1})}_{\text{FINITE GEOM. SERIES}} = p\,\frac{(1-q^x)}{1-q} = (1-q^x)$

NOW $p = 1/2 \Rightarrow q = 1/2 \quad F(x) = 1 - \left(\frac{1}{2}\right)^x$



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3) BINOMIAL $n = 15$ $\quad X$ = NO. QUEST. ANSWERED CORRECTLY

CORRECT = SUCCESS $\quad P(S) = 1/5 \quad P(F) = 4/5$

2/3 OF 15 = 10 QUEST $\quad$ FIND $P(X \ge 10) = 1 - P(X < 10) = 1 - P(X \le 9)$

$= 1 - \sum_{x=0}^{9} \binom{15}{x} (.2)^x (.8)^{15-x}$ → USE TABLES $= 1 - .9999 = .0001$

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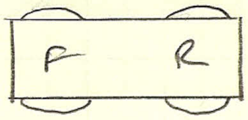
4) $p = .2$ NEG. BINOMIAL $r = 3$ $\quad X$ = NO. TRIAL ON WHICH rTH SUCCESS OCCURS

$P(X = 7) = \binom{7-1}{3-1} p^3 (1-p)^{7-3} = \binom{6}{2} (.2)^3 (.8)^4$

$= \frac{6!}{2!\,4!} (.2)^3 (.8)^4 = .049152$

5) 

$\lambda_1 = 2$ $\lambda_2 = 1.5$

$X_1 \equiv$ NO. FRONT TIRES REPLACED

$X_2 \equiv$ NO. REAR TIRES REPLACED

$A \equiv$ GET 2 TIRES REPLACED

POISSON $\equiv$ INDEP

$$P(A) = P(X_1=0 \cap X_2=2) + P(X_1=1 \cap X_2=1) + P(X_1=2 \cap X_2=0)$$
$$= P(X_1=0)P(X_2=2) + P(X_1=1)P(X_2=1) + P(X_1=2)P(X_2=0)$$
$$= \left(\frac{e^{-2}2^0}{0!}\right)\left(\frac{e^{-1.5}(1.5)^2}{2!}\right) + \left(\frac{e^{-2}2^1}{1!}\right)\left(\frac{e^{-1.5}(1.5)^1}{1!}\right) + \left(\frac{e^{-2}2^2}{2!}\right)\left(\frac{e^{-1.5}(1.5)^0}{0!}\right)$$
$$= e^{-3.5}\left(\frac{(1.5)^2}{2} + 3 + 2\right) = .1849$$

6) HYPERGEOMETRIC X = NO. A.A. JURORS

$N = 20$ $N_1 = 8$

$n = 6$ $N_2 = 12$

$$P(X \le 1) = P(X=0) + P(X=1) = \frac{\binom{8}{0}\binom{12}{6}}{\binom{20}{6}} + \frac{\binom{8}{1}\binom{12}{5}}{\binom{20}{6}}$$
$$= .023839003 + .163467492 = .1873$$

SIG. PROB. OF GETTING NONE OR ONE AA JUROR. NOT UNUSUAL $\Rightarrow$ SELECTION PROCEDURE NOT SUSPICIOUS.

7) a) X = FAMILY SIZE $x = 0, 1, 2, \ldots$ $P(\text{FEMALE}) = P(\text{MALE}) = .5$

Y = NO FEMALES $y = 0, 1, 2, \ldots$ FEMALE $\equiv$ SUCCESS

BINOMIAL $P(Y=k \mid X=n) = \binom{n}{k}(.5)^k(.5)^{n-k} = \binom{n}{k}(.5)^n$

b) $P(X=n) = \dfrac{e^{-2.25}(2.25)^n}{n!}$ $A \equiv$ HAVE EXACTLY 2 BOYS AND 2 GIRLS

$$P(A) = P(Y=2 \cap X=4) = P(Y=2 \mid X=4)\,P(X=4)$$
$$= \binom{4}{2}(.5)^4 \cdot \frac{e^{-2.25}(2.25)^4}{4!} = 6 \cdot \frac{1}{16} \cdot e^{-2.25}\frac{(2.25)^4}{24} = .0422$$

c) $B \equiv$ HAVE CHILDREN OF BOTH SEXES

$$P(B) = P(Y=1 \cap X \ge 2) = P(Y=1 \mid X \ge 2)\,P(X \ge 2)$$
$$= \sum_{x=2}^{\infty} P(Y=1 \mid X=x)\,P(X=x) = \sum_{x=2}^{\infty} \binom{x}{1}\left(\frac{1}{2}\right)^x \cdot \frac{e^{-2.25}(2.25)^x}{x!}$$