Prob + Stat

Post class Problems - Nov 16

4.1

3) $Y = X_1 - X_2$ $\quad \mu_Y = \mu_1 - \mu_2$ $\quad \sigma_Y^2 = \sigma_1^2 + \sigma_2^2$

$= 3 - 6$ $\quad = 9 + 16$

$= -3$ $\quad = 25$

$\therefore Y = N(\mu = -3, \sigma = 5)$

$\sigma_Y = \sqrt{25}$

$= 5$

$$P(-10 < Y < 5) = P\left(\frac{-10-\mu}{\sigma} < Z < \frac{5-\mu}{\sigma}\right)$$

$$= P\left(\frac{10-(-3)}{5} < Z < \frac{5-(-3)}{5}\right)$$

$$= P(-1.4 < Z < 1.6)$$

$$= \Phi(1.6) - \Phi(-1.4)$$

$$= \Phi(1.6) - (1 - \Phi(1.4))$$

$$= 0.9452 - (1 - 0.9192)$$

$$= 0.8644$$

$$T = \frac{\bar{X} - \mu}{S/\sqrt{16}}$$

5) a) $P\left(-d < \frac{\bar{X} - \mu}{S/\sqrt{16}} < d\right) = 0.95$

$$P(T < d) - P(T < -d) = 0.95$$

$$P(T < d) - (1 - P(T < d)) = 0.95$$

$$2P(T < d) - 1 = 0.95$$

$$P(T < d) = 0.975$$

Use table VI and notice that $r = n - 1 = 16 - 1 = 15$

$d = 2.131$

b) $P\left(-2.131 < \frac{\bar{X} - \mu}{s/\sqrt{16}} < 2.131\right) = 0.95$

$P\left(\frac{s}{4}(-2.131) < \bar{X} - \mu < \frac{s}{4}(2.131)\right) = 0.95$

$P\left(\bar{X} - \frac{s}{4}(-2.131) > \mu > \bar{X} - \frac{s}{4}(2.131)\right) = 0.95$

$P\left(\bar{X} - \left(\frac{s}{4}\right)2.131 < \mu < \bar{X} + \left(\frac{s}{4}\right)(2.131)\right) = 0.95$

Swapped the two inequalities

$P(u(\bar{X}, s) < \mu < v(\bar{X}, s)) = 0.95$

$u(\bar{X}, s) = \bar{X} - \left(\frac{s}{4}\right)2.131$

$v(\bar{X}, s) = \bar{X} + \left(\frac{s}{4}\right)2.131$

7) a) $P(-d < T < d) = 0.90$

$P(T<d) - P(T<-d) = 0.90$

$P(T<d) - (1 - P(T<d)) = 0.90$

$2P(T<d) - 1 = 0.90$

$P(T<d) = 0.95$

$r = m + n - 2$

$r = 8 + 10 - 2$

$r = 16$

Using Table VI

$d = 1.746$

b) $$T = \frac{\dfrac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\sqrt{\sigma^2(1/n + 1/m)}}}{\sqrt{\dfrac{[(n-1)s_X^2 + (m-1)s_Y^2]/\sigma^2}{n+m-2}}}$$

(from 4.1-6 (c))

(notice that the σ^2's cancel out

$$T = \frac{\dfrac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{(1/8 + 1/10)}}{\sqrt{\dfrac{7s_X^2 + 9s_Y^2}{16}}}$$

and $d = 1.746$

$$\therefore P(-d < T < d) = P\left(\bar{X} - \bar{Y} - 1.746\sqrt{\frac{7s_X^2 + 9s_Y^2}{16}}\sqrt{\frac{1}{8} + \frac{1}{10}} < T < \bar{X} - \bar{Y} + 1.746\sqrt{\frac{7s_X^2 + 9s_Y^2}{16}}\sqrt{\frac{1}{8} + \frac{1}{10}}\right)$$

4.2

3) $\sigma^2 = \sigma^2 \left[\dfrac{\sum_{i=1}^{n} (X_i - \mu)^2}{\sum_{i=1}^{n} (X_i - \mu)^2} \right]$

$= \sum_{i=1}^{n} (X_i = \mu)^2$

9) a) $\bar{x} = \frac{1}{10} \sum_{i=1}^{10} x_i$

$= 3.58$

b) $s = \sqrt{Var[X]}$

$= 0.512$

c) One sided C.I.:

$[0, \bar{x} + t_\alpha(r)(\sigma/\sqrt{n})]$

10 observations $r = 10 - 1 = 9$

lowest possible μ is 0

$= [0, 3.58 + t_{0.05}(9)(0.512/\sqrt{10})]$

$= [0, 3.877]$

look in table $\underline{VI}$

11) a) $\bar{x} = 25.475$

$s = 2.4935$

b) $[\bar{x} - t_\alpha(r)(\sigma/\sqrt{n}), \infty]$

$[25.475 - t_{0.01}(19)(2.4935/\sqrt{20}), \infty)$

$[24.059, \infty)$

13) a) $X - Y = N(\mu = \mu_x - \mu_y, \sigma = \sqrt{\sigma_x^2 + \sigma_y^2})$

CI: $[\bar{x} - t_{\alpha/2}(n-1)(s/\sqrt{n}), \bar{x} + t_{\alpha/2}(n-1)(s/\sqrt{n})]$

$= [\bar{x}_x - \bar{x}_Y - t_{0.05/2}(12+16-2)(\frac{\sqrt{s_x^2 + s_Y^2}}{v}$

13. a) $S_p = \sqrt{\dfrac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2}}$ $t_0 = t_{\alpha/2}(n+m-2)$

CI: $\left[\bar{x} - \bar{y} - t_0 s_p\sqrt{\frac{1}{n} + \frac{1}{m}},\ \bar{x} - \bar{y} + t_0 s_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right]$

$[-115.480, 129.105]$

c) Since 0 (i.e. no difference) is w/in the confidence interval, we cannot conclude that there is a difference in butterfat production.

15 a) $d = \sigma_X^2/\sigma_Y^2$ $\therefore \dfrac{(\bar{X}-\bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{d\sigma_Y^2/n + \sigma_Y^2/m}} = \dfrac{(\bar{X}-\bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\sigma_X^2/n + \sigma_Y^2/m}}$

which is $N(0,1)$

b) $\dfrac{(n-1)S_X^2}{d\sigma_Y^2} + \dfrac{(m-1)S_Y^2}{\sigma_Y^2} = \dfrac{(n-1)S_X^2}{\sigma_X^2} + \dfrac{(m-1)S_Y^2}{\sigma_Y^2}$

$= \chi(n-1) + \chi(m-1)$

$= \chi(n+m-2)$

c) Sample means and variances are independant (See page 165)

d) $\bar{x} - \bar{y} \pm t_{\alpha/2}(n+m-2)\sqrt{\dfrac{(n-1)S_X^2/d + (m-1)S_Y^2}{n+m-2}\left(\dfrac{d}{n} + \dfrac{1}{m}\right)}$