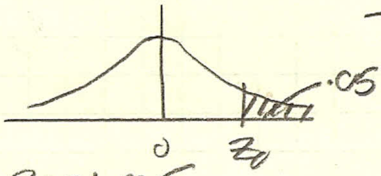
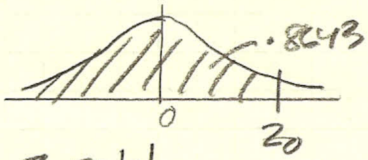
DAY 8: INCLASS SOLS

1) a)



$z_0 = 1.645$

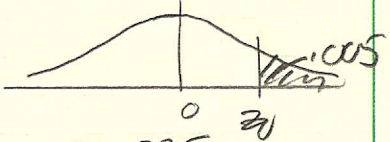
b)



$z_0 = 1.1$

c) $P(-z_0 < z < z_0) = .9$
$\Rightarrow P(z > z_0) = .05$
SO $z_0 = 1.645$

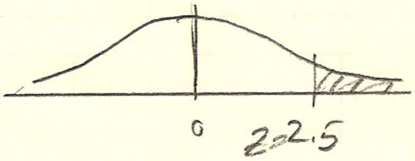
d) $P(-z_0 < z < z_0) = .99$
$\Rightarrow P(z > z_0) = .005$



$z_0 = 2.575$

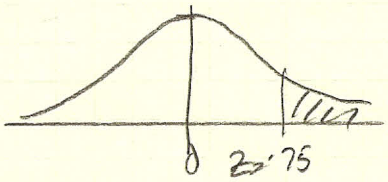
2) $X \equiv$ AMT. SPENT ON MAINT./REPAIR $\quad X \sim N(\mu, \sigma^2) = N(400, (20)^2)$

WANT $P(X > 450) = P\left(\frac{X-\mu}{\sigma} > \frac{450-400}{20}\right) = P(Z > 2.5)$



$= .0062$

3) $X = \text{GPA} \quad X \sim N(\mu, \sigma^2) = N(2.4, (0.8)^2)$



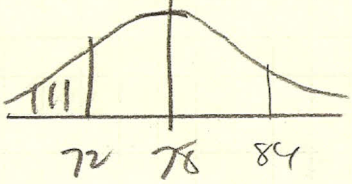
$P(X > 3) = P\left(\frac{X-\mu}{\sigma} > \frac{3-2.4}{.8}\right) = P(Z > .75) = .2266$

SO 22.66% OF STUDENTS HAVE GPA IN EXCESS OF 3.0

4) $X \sim N(\mu, \sigma^2) \quad A = LW = 3X \cdot X = 3X^2 \quad E(A) = E(3X^2) = 3E(X^2)$

SINCE $\sigma^2 = E(X^2) - \mu^2 \Rightarrow E(X^2) = \sigma^2 + \mu^2 \quad = 3(\sigma^2 + \mu^2)$

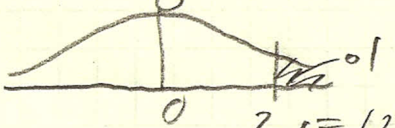
5) a)



$P(X > 72) = 1 - P(X < 72)$
$= 1 - P(X > 84)$
$= 1 - P(Z > 1) = 1 - .1587 = .8413$

$z = \frac{84-78}{6} = 1$

b) $z = \frac{X-\mu}{\sigma} \Rightarrow X = z\sigma + \mu$



$z_{.1} = 1.285$

$X = (1.285)6 + 78 = 85.71$

$P(X > 85.71) = .1$

c)

$z = .58 \quad X = (.58)6 + 78 = 81.48$

CUTOFF = 81.48%

6) a) $L(\theta)=\prod_{i=1}^{n} f(x_i)=\prod_{i=1}^{n}\frac{1}{\theta^2}x_i e^{-x_i/\theta} = \left(\frac{1}{\theta^2}\right)^n\left(\prod_{i=1}^{n}x_i\right)e^{-\frac{\sum_{i=1}^{n}x_i}{\theta}} = \theta^{-2n}\left(\prod_{i=1}^{n}x_i\right)e^{-\frac{n\bar{x}}{\theta}}$

$\ell(\theta)=\ln(L(\theta)) = -2n\ln(\theta)+\ln\left(\prod_{i=1}^{n}x_i\right)-\frac{n\bar{x}}{\theta}$

b) $\frac{d\ell}{d\theta} = \frac{-2n}{\theta} - n\bar{x}\left(-\frac{1}{\theta^2}\right)=0$ $\quad \frac{n\bar{x}}{\theta^2}=\frac{2n}{\theta} \Rightarrow n\bar{x}\theta = 2n\theta^2$ $\quad \theta(2\theta-\bar{x})=0$

$2n\theta^2-n\bar{x}\theta=0$ $\quad \theta=0$

$2\theta^2-\bar{x}\theta=0$ $\quad$ or $\quad \theta=\frac{\bar{x}}{2}$

$\frac{d^2\ell}{d\theta^2} = -2n\left(\frac{-1}{\theta^2}\right)+n\bar{x}\left(\frac{-2}{\theta^3}\right)$

$= \frac{2n}{\theta^2}-\frac{2n\bar{x}}{\theta^3}\Big|_{\theta=\frac{\bar{x}}{2}} = \frac{2n\cdot 4}{\bar{x}^2}-\frac{2n\bar{x}\cdot 8}{\bar{x}^3} = \frac{8n\bar{x}-16n\bar{x}}{\bar{x}^3} = \frac{-8n}{\bar{x}^2}<0$ always

$\therefore \hat{\theta}=\frac{\bar{x}}{2}$ IS THE M.L.E.

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7) $L(\theta)=\prod_{i=1}^{n} f(x_i)=\prod_{i=1}^{n}(\theta+1)x_i^{\theta} = (\theta+1)^n\left(\prod_{i=1}^{n}x_i\right)^{\theta}$ SO $\ell(\theta)=\ln(\theta)$

$= n\ln(\theta+1)+\theta\ln\left(\prod x_i\right)$

$\frac{d\ell}{d\theta}=\frac{n}{\theta+1}+\ln\left(\prod_{i=1}^{n}x_i\right)=0 \Rightarrow \frac{n}{\theta+1}=-\ln\left(\prod x_i\right)$ $\quad \theta+1=\frac{n}{-\ln(\prod x_i)}$

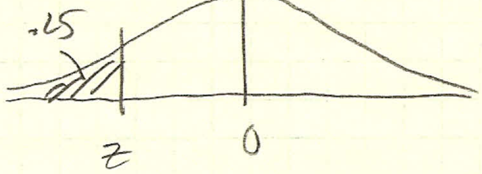
$\theta=\frac{n}{-\ln(\prod x_i)}-1$

$\frac{d^2\ell}{d\theta^2}=n(-1)(\theta+1)^{-2}$

$= \frac{-n}{(\theta+1)^2}<0$ ALWAYS!

THUS $\hat{\theta}=-\left(1+\frac{n}{\ln(\prod x_i)}\right)$ IS THE M.L.E.

#5d)



$Z=-.675$

$X=6(-.675)+78=73.95$

$73.95+5=78.95$

$P(X>78.95)=P\left(Z>\frac{78.95-78}{6}\right)=P(Z>.1583)=.4364$

SO 43.64%