Block Print Your Name: ANSWERS!

Probability and Statistics, Fall 2009, Quiz 1
Sections 1.1-1.3, Introductory Probability

- No resources are allowed, except for a calculator/computer for basic arithmetic; do not use any pre-programmed formulas.
- Explain your answers in order on additional sheets of paper as needed.
- There is a strict 90 minute limit for this quiz. Set an alarm. (A goal should be to finish in 60 minutes.)
  Initial Quiz Download/View (Time and Date):
  End of Quiz (Time and Date):

1. A bowl has 9 green and 7 red M & Ms. Three pieces of candy are randomly selected in succession (i.e., one after the other) without replacement.
   (a) Write out the sample or outcome space for this experiment.
   $S = \{GGG, RRR, RGG, GRG, GGR, RRG, RGR, GRR\}$
   (b) Let the event $A = \{\text{At least two pieces are green}\}$. What is $P(A)$? Carefully explain your reasoning, showing all work.
   This problem can be done by looking at the outcome space and assigning probabilities to each event in the outcome space. There are 4 events such that at least two pieces are green, so we must consider these 4 events: GGG, GRG, GGR, and RGG. The probability of the first event listed here is (9/16)(8/15)(7/14), since we have 9 total green candies to begin and we are choosing in a specified order without replacement. The probability for the second event is (9/16)(7/15)(8/14) for similar logic. The next two events have probabilities (9/16)(8/15)(7/14) and (7/16)(9/15)(8/14), respectively. Notice that although the individual terms in each product vary, the last three analyzed events have probabilities that are numerically equivalent, and common sense says that the probabilities of events GRG, GGR, and RGG should be equal. (The probability of the first event is also numerically equal to the others, but for a different reason!) Adding up the probabilities of these 4 events gives the desired $P(A)$, which is 0.6 when simplified.

2. A committee of 50 senators is randomly selected from the U.S. senate (which includes 2 senators from each of the 50 states). **NOTE: The wrong quiz file was uploaded that did not have this parenthetical remark. Due to this mistake, and due to the fact that several students did not know the composition of the senate, we could not grade this problem.**
   (a) What is the probability that Virginia is not represented on this committee? Carefully explain your reasoning, showing all work.
   The total number of ways to select 50 random senators from the group of 100 total senators is $\binom{100}{50}$. This will be my denominator. The total number of ways to select 50 random senators without including Virginia is $\binom{98}{50}$ because of the 100 total senators, I cannot choose either of the two from VA. So, the probability of the requested event is $\binom{98}{50}/\binom{100}{50}$. These numbers might be too big to directly use your calculator, but by using the definition of "choose" we can simplify to $(50 * 49)/(100 * 99)$ which gives approximately 0.247.
   (b) What is the probability that every state is represented on the committee? Carefully explain your reasoning, showing all work.
   The denominator will be the same as above. For the numerator, we must count the number of ways to choose 1 representative from each of the 50 states. For each of the 50 states, we have 2 choices (senator 1 or senator 2 from a given state). So, there are 2^{50} ways to pick one senator from each state. The final answer is $2^{50}/\binom{100}{50}$ (which is very small).

3. A lot contains 15 items from supply line A and 25 items from supply line B. Two items are selected randomly, in succession, without replacement, from the lot of 40. Let A be the event that the **first** item selected is from supply line A; let B be the event that the **second** item is selected from supply line B. Determine the following, explaining your reasoning and **using a calculator to obtain final numerical answers**:

(a) $P(A) = 15/40 = 0.375$ because there are 15 possible ways to obtain the first item from line A and 40 ways to obtain any item.

(b) $P(B|A)$. Given that A has occurred, we know that one item from A has been removed and we still have 25 items available from B and 39 total items available. So, the probability of B given that A has occurred is $25/39 = 0.641$.

(c) $P(A \bigcap B)$. Here we need to get an item from A first AND an item from B second. The first can happen with probability 15/40; the second can happen with probability 25/39 since we have 25 possible items from B and 39 total items remaining. Mathematically, $P(A \bigcap B) = P(A)P(B|A)$. We multiply these probabilities $(15/40)(25/39) = 0.240$.

4. Draw cards from a standard deck successively at random and without replacement. What is the probability that the sixth diamond appears on exactly the tenth draw from the deck? Carefully explain your reasoning, showing all work.

   Let's answer this in two different ways.

   Answer 1: If we know the sixth diamond must appear exactly on the tenth draw, this means we must have 5 diamonds appearing somewhere within the first 9 draws and a 6th diamond on the 10th draw. There are $\binom{9}{5}$ ways to place 5 diamonds within 9 draws, b/c we simply pick the places where they will appear. This gives the number of ways we can place the diamonds (e.g., the number of relevant events), but now we must consider the probability of each of these events. For each of these events (which look like strings of length 9 consisting of 5 diamonds and 4 non-diamonds, followed by a diamond), we must compute the probability of getting such a string. The probability of any of these strings is the same, so we can consider without loss of generality the probability of first getting 5 diamonds in a row when choosing without replacement, which gives (13/52)(12/51)(11/50)(10/49)(9/48) because we look at the changing ratios of number of available diamonds to number of available cards. Then, the probability of getting 4 non-diamonds in a row as the next cards when choosing without replacement is (39/47)(38/46)(37/45)(36/44), by looking at the changing ratios of number of available non-diamonds to number of available cards. Finally, we can get a 6th diamond in the 10th spot with probability (13-5)/(52-9) = 8/43, based on the number of remaining diamonds and the number of remaining cards. We must use the product rule for the probability of getting a string of length 9 with 5 diamonds and 4 non-diamonds, followed by a diamond: (13/52)(12/51)(11/50)(10/49)(9/48)(39/47)(38/46)(37/45)(36/44)(8/43). Our final answer multiplies this probability by the number of possible events:

$$\binom{9}{5}(13/52)(12/51)(11/50)(10/49)(9/48)(39/47)(38/46)(37/45)(36/44)(8/43).$$

   Answer 2: Let A be the event of 5 diamonds appearing in the first 9 cards drawn. Let B be the event of a diamond appearing on the 10th draw. We need to compute $P(A \bigcap B) = P(A)P(B|A)$. For $P(A)$, count the total number of ways to pick 5 diamonds out of 9 cards in $\binom{13}{5}\binom{39}{4}$ ways, since we pick 5 diamonds from 13 possible diamonds and 4 non-diamonds from 39 possible non-diamonds. The total number of way to pick any 9 cards is $\binom{52}{9}$ so $P(A) = \binom{13}{5}\binom{39}{4}/\binom{52}{9}$. Now, $P(B|A)$ must compute the probability of getting a diamond on the 10th draw after getting 5 diamonds in the previous 9 cards. There are 13-5 = 8 diamonds remaining and 52 - 9 = 43 cards remaining, so $P(B|A) = 8/43$. Our final answer is $(8/43)\binom{13}{5}\binom{39}{4}/\binom{52}{9}$.

   The two answers are numerically equivalent.

5. If $P(X|Y) = 1$, what does this imply about the relationship between events X and Y? In your well-explained answer, include any appropriate mathematical formulas, a Venn diagram, and an example.

   If $P(X|Y) = 1$, then this means that given that Y has occurred, X must occur. This implies that Y must be a subset of X (including the possibility that $Y = X$) since Y occurring implies that X is also occurring. Mathematically, we know that $P(X|Y) = P(X \bigcap Y)/P(Y)$, so $P(X|Y) = 1$ implies that $P(X \bigcap Y) = P(Y)$. This equality says that looking only at the part of Y that intersects with X does not differ from looking at all of Y; this again confirms that Y must be a subset of X. As an example consider rolling a die and X is the event of getting an even number and Y is the event of getting a 2. Whenever Y has happened, we have a 2, so we certainly have an even number with probability 1. For a Venn diagram, you should have a diagram where a circle representing Y is contained within a circle representing X, both of which are inside a rectangular sample space.