

Probability + Statistics

Nov 19 - In class Probs

$$\begin{aligned} 1. a) P(\text{type I error}) &= P(\text{reject null hypothesis} / \text{null hypothesis is true}) \\ &= P(\bar{X} > 25 \text{ or } \bar{X} < 15 \mid \mu = 20) \\ &= 2P(\bar{X} > 25 \mid \mu = 20) \quad \text{b/c } X \text{ is symmetric about } 20 \\ &= 2P\left(\frac{\bar{X} - 20}{\sqrt{25}/\sqrt{9}} > 3\right) \\ &= 2P(Z > 3) \\ &= 2(0.0013) \\ &= 0.0026 \end{aligned}$$

$$\begin{aligned} b) P(\text{type II}) &= P(15 < \bar{X} < 25 \mid \mu = 23) \\ &= P\left(-3 < \frac{\bar{X} - 20}{5/3} < 3 \mid \mu = 23\right) \\ &= P\left(-3 - \frac{20 - 23}{5/3} < \frac{\bar{X} - 23}{5/3} < 3 - \frac{20 - 23}{5/3}\right) \\ &= P(-4.8 < Z < 1.2) \\ &= \Phi(1.2) - \Phi(-4.8) \\ &= \Phi(1.2) - (1 - \Phi(4.8)) \\ &= 0.5478 - (1 - 0.6844) \\ &= 0.2322 \end{aligned}$$

$$2. H_0: \sigma = 0.05$$

$$H_a: \sigma < 0.05$$

Using tests of statistical hypotheses can only find conclusive evidence that alternative hypothesis is true. \therefore Make H_a what you want to ~~then~~ prove.

Type I: We conclude that $\sigma < 0.05$ but σ actually is 0.05

Type II: We conclude that $\sigma = 0.05$ but σ is actually < 0.05

$$\begin{array}{lll}
 3. & H_0: \mu = 48 & n = 12 & \alpha = 0.1 \\
 & H_a: \mu \neq 48 & \bar{x} = 47.1 & \\
 & (\text{2 sided test}) & s^2 = 4.7 &
 \end{array}$$

a) Reject H_0 if

$$\begin{aligned}
 -t_{(n-1)\alpha/2} &> \frac{\bar{x} - \mu}{s/\sqrt{n}} \quad \text{or} \quad t_{(n-1)\alpha/2} < \frac{\bar{x} - \mu}{s/\sqrt{n}} \\
 -1.796 &> \frac{47.1 - 48}{\sqrt{4.7}/\sqrt{12}} \quad \text{or} \quad 1.796 < \frac{47.1 - 48}{\sqrt{4.7}/\sqrt{12}} \\
 -1.796 &> -1.438 \quad \text{or} \quad 1.796 < -1.438
 \end{aligned}$$

Neither are true, fail to reject H_0

$$\begin{aligned}
 b) & \left[\bar{x} - t_{\alpha/2}(n-1) \left(\frac{s}{\sqrt{n}} \right), \bar{x} + t_{\alpha/2}(n-1) \left(\frac{s}{\sqrt{n}} \right) \right] \\
 & \left[47.1 - t_{0.05}(12-1) \left(\sqrt{\frac{4.7}{12}} \right), 47.1 + t_{0.05}(12-1) \left(\sqrt{\frac{4.7}{12}} \right) \right] \\
 & [45.976, 48.224]
 \end{aligned}$$

If we were to repeat the experiment a large number of times and calculate the confidence interval each time, the true mean would be in 90% of the intervals.

4. a) $H_0: \sigma = 0.25$ $S = 0.37$
 $H_1: \sigma \neq 0.25$ $n = 51$
 \therefore 2-sided test $\alpha = 0.05$

Assuming S is normally distributed

Reject H_0 if:

$$\sigma^2 < \frac{(n-1)s^2}{\chi^2_{\alpha/2}(n-1)} \quad \text{or} \quad \sigma^2 > \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}(n-1)}$$

$$(0.25)^2 < \frac{(51-1)(0.37)^2}{71.42} \quad \text{or} \quad (0.25)^2 > \frac{(51-1)(0.37)^2}{32.36}$$

$$0.0625 < 0.0958 \quad \text{or} \quad 0.0625 > 0.2115$$

$\nearrow \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\alpha/2}(n-1)}$ So reject H_0 !

b) Construct a 2-sided 95% CI for $\alpha = 0.05$.

If ~~the~~ $\sigma = 0.25$ is outside the CI, then reject H_0 .

5. a) Reject H_0 if

$$t \geq t_{\alpha}(n-1)$$

$$3.2 \geq t_{0.05}(15-1)$$

$$3.2 \geq 1.761$$

reject null hypothesis

b)

$$t \geq t_{\alpha}(n-1)$$

$$1.8 \geq t_{0.01}(9-1)$$

$$1.8 \geq 2.896$$

fail to reject null hypothesis

c)

$$t \geq t_{\alpha}(n-1)$$

$$-0.2 \geq t_{\alpha}(24-1)$$

$$-0.2 \geq t_{\alpha}(23)$$

$t_{\alpha}(23)$ is always positive, so

fail to reject null hypothesis