Your name(s):

# Day 8: Normal Distribution, Maximum Likelihood Estimators
## Sec 3.4-3.5

1. If $Z$ is a standard normal random variable, find the value of $z_0$ such that
   (a) $P(Z > z_0) = 0.5$
   (b) $P(Z < z_0) = 0.8643$
   (c) $P(-z_0 < Z < z_0) = 0.9$
   (d) $P(-z_0 < Z < z_0) = 0.99$

2. The weekly amount of money spent on maintenance and repairs by a company was observed, over a long period of time, to be approximately normally distributed with mean \$400 and standard deviation \$20. If \$450 is budgeted for next week, what is the probability that the actual costs will exceed the budgeted amount?

3. The GPAs of a large population of college students are approximately normally distributed with mean 2.4 and standard deviation 0.8. What fraction of students possess GPA in excess of 3.0?

4. Assume $X$ is normally distributed with mean $\mu$ and standard deviation $\sigma$. After observing a value for $X$, a mathematician constructs a rectangle with length $L = |X|$ and width $W = 3|X|$. Let $A$ denote the area of the resulting rectangle. What is $E(A)$?

5. Scores on an examination are assumed to be normally distributed with mean 78 and variance 36.
   (a) What is the probability that a person taking the exam scores higher than 72?
   (b) Suppose that students scoring in the top 10% of this distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A grade?
   (c) What must be the cutoff point for passing the exam if the examiner wants only the top 28.1% of all scores to be passing?
   (d) Approximately what proportion of students have scores 5 or more points above the score that cuts off the lowest 25%?

6. A certain type of electrical component has a lifetime $Y$ (in hours) with probability density function given by

$$f(y) = \begin{cases} \frac{1}{\theta^2} y e^{-y/\theta} & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What is the likelihood function for an independent random sample $Y_1, Y_2, \ldots Y_n$ from this distribution? Write an expression for the log-likelihood function.
   (b) Derive the maximum likelihood estimator $\hat{\theta}$ for parameter $\theta$.

7. Let $X_1, X_2, \ldots, X_n$ denote an independent random sample from the following probability distribution

$$f(x) = \begin{cases} (\theta+1)x^{\theta} & 0 \leq x \leq 1 \ , \ \theta > -1 \\ 0 & \text{otherwise} \end{cases}$$

Find the maximum likelihood estimator for the parameter $\theta$.