Your name(s):

Day 4: Discrete Probability Distributions and Expectation
Sec 2.1-2.2

1. An E! team estimates that when it introduces its new product to the public, it will be very successful (VS) with a probability of 0.6, moderately successful with a probability of 0.3 (MS), and not successful (NS) with a probability of 0.1. The estimated yearly profit associated with being VS is \$15mil; estimated yearly profit for MS is \$5 mil; and estimated yearly loss for NS is \$500,000. Let X be the yearly profit of the new product. Determine the PMF for X.

2. A second E! team is confident their product with earn \$7 million per year. Find the expected profits for the first and second teams. Before you pick which team to back, you should consider the variability or possible deviation of the expected profits. Describe the variance for both teams' profits. What units are used? What alternative measure would give you a more easily interpreted measure of variability?

3. Suppose $O = \{1, 2, 3, \ldots\}$ and assign probabilities to the outcomes as follows:
$$p_k = \Pr(\text{Outcome is } k) = (1-p)^k p,\ 0 < p < 1,\ k = 1, 2, 3, \ldots.$$
Verify that p_k is a PMF on the outcome space O. This probability distribution is known as the *geometric* PMF.

4. Here is a problem from first-year mechanics. Suppose that point masses are placed along the positive x-axis in the following way: a mass of $m_1 = \frac{1}{2}$ is placed at $x = 1$, a mass of $m_2 = \frac{1}{4}$ is placed at $x = 2$ and so on—in general a mass of $m_k = \frac{1}{2^k}$ is placed at $x = k$. What is the moment of this distribution of mass about the y-axis? Where is the center of mass of this distribution?

5. In a gambling game, a person draws a single card from an ordinary 52-card playing deck. A person is paid \$15 for drawing a jack or queen and \$5 for drawing a king or ace. If they draw any other card, they must pay $\$x$. Derive an expression for the expected gain/loss as a function of x. How large can x be for the game to be profitable to the person playing?

6. Five balls, numbered 1, 2, 3, 4 and 5, are placed in an urn. Two balls are randomly selected without replacement from the five and their numbers are noted. Find the probability distribution for the *largest* of the two sampled numbers. Compute the mean and the variance

7. Let X be $b(2, p)$ and Y be $b(4, p)$ (binomial RVs with different numbers of trials). If $P(X \geq 1) = \frac{5}{9}$, find $P(Y \geq 1)$.