Your name(s):

Day 5: Expectations, Special Discrete Distributions
Sec 2.2-2.3

1. A new secretary has been given n computer passwords, only one of which will permit access to a computer file. Because the secretary has no idea which password is correct, he chooses one of the passwords at random and tries it. If the password is incorrect, he discards it and randomly selects another password from the ones remaining, proceeding in this manner until he finds the correct password.

   (a) What is the probability that he obtains a correct password on the first try?

   (b) What is the probability that he obtains a correct password on the second try? The third try?

   (c) A security system has been set up so that if three incorrect passwords are tried before the correct one, the computer file is locked and access to it denied. If $n = 7$, what is the probability that the secretary will gain access to the file?

   (d) Find the mean and variance for the random variable X, which is the number of trials required to open the file.

2. Consider a discrete random variable X with a geometric probability distribution:

   $$\Pr(X = x) = (1-p)^{x-1}p,\ 0 < p < 1,\ k = 1, 2, 3, \ldots.$$

   Determine the CDF for this random variable. Sketch a graph of the CDF when $p = 1/2$.

3. A multiple-choice exam has 15 questions, each with five possible answers, only one of which is correct. Suppose that one of the students who takes the exam answers each of the questions with an independent random guess. What is the probability that he answers at least two-thirds of the questions correctly?

4. A geological study indicates that the drilling of an oil exploratory well should strike oil with probability 0.2. An oil company wishes to explore a new oil field. How likely is it that the company strikes oil for the 3rd time on the 7th well it drills.

5. A particular type of tire does not hold up well for vehicles on unpaved roads in a rural Western state. Vehicles get their front tires replaced on an average of two per year, while rear tires get replaced on an average of one and a half per year. Assuming the replacement of front tires is independent of replacement of rear tires, what is the probability that a random vehicle has two of its tires replaced within a given year?

6. A jury of 6 persons was selected from a group of 20 potential jurors, of whom 8 are African American and 12 were Caucasian. The jury was supposedly randomly selected, but it contained only 1 African American member. Do you have reason to doubt the randomness of the selection? Justify your answer by computing an appropriate probability.

7. Suppose that each birth of a baby is equally likely to be a male or a female.

   (a) Given that a family has exactly n children, $n = 1, 2, \ldots$, what is the probability that exactly k of them are female, $k = 0, 1, \ldots, n$?

(b) As you know from experience, not all families have the same number of children. A good approximation to the distribution of family size is the Poisson distribution, with mean 2.25. Thus the probability that a family will have exactly n children is $(2.25)^n e^{-2.25}/n!$. What fraction of families will have exactly two boys and two girls?

(c) What is the probability that a family will have children of both sexes?