Integration Rules and Properties

Integral Properties:

1. Switching limits:
$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$
2. Definite integral over a point:
$$\int_a^a f(x)\,dx = 0$$
3. Definite integral of a constant:
$$\int_a^b c\,dx = c(b-a)$$
4. Integral of a sum/difference:
$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$
5. Integral of a constant times a function:
$$\int cf(x)\,dx = c\int f(x)\,dx$$

Basic Integration Rules:

1. Power Rule
$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \text{ for } n \neq -1$$
2. Exponential Function
$$\int e^x\,dx = e^x + C$$
3. General Exponential Function
$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$
4. Natural Logarithm Function
$$\int \frac{1}{x}\,dx = \ln x + C$$
5. Trigonometric Functions

$$\int \cos x\,dx = \sin x + C \qquad \int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

6. Integration of Inverse Trigonometric Functions

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C \quad \int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

U-substitution Integration Rules:

1. Power Rule

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C \text{ for } n \neq -1$$

2. Exponential Function

$$\int e^u\,du = e^u + C$$

3. General Exponential Function

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

4. Natural Logarithm Function

$$\int \frac{1}{u}\,du = \ln u + C$$

5. Trigonometric Functions

$$\int \cos u\,du = \sin u + C \qquad \int \sin u\,du = -\cos u + C$$

$$\int \sec^2 u\,du = \tan u + C \qquad \int \csc u \cot u\,du = -\csc u + C$$

$$\int \sec u \tan u\,du = \sec u + C \quad \int \csc^2 u\,du = -\cot u + C$$

6. Integration of Inverse Trigonometric Functions

$$\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1} u + C \quad \int \frac{1}{1+u^2}\,du = \tan^{-1} u + C$$