MATH 363 – Sample Second Exam Questions

April 25, 2011

The following are questions of the type that will appear on the second hour exam. The exam will have roughly the same number of parts, but may be organized differently.

1. True or false, with explanation or counterexample.

   (a) If a topological space X is path connected, then $\pi_1(X, p) = \{e\}$.

   (b) If a topological space X is contractible, then $\pi_1(X, p) = \{e\}$.

   (c) If X is contractible and a topological group G acts on X, then $\pi_1(X/G, p) = \{e\}$.

2. Let $X \subset \mathbb{R}^2$ be defined by

$$X = \{(x, y) : (x + 1)^2 + y^2 = 1\} \cup \{(x, y) : (x - 1)^2 + y^2 = 1\}.$$

   (See Figure 1.) Assume that X has the induced topology from $\mathbb{R}^2$.

   (a) Find a set of generators for the fundamental group of X.

   (b) Using your generators, what form does a typical element of the fundamental group take?

   (c) Is the fundamental group of X abelian? Explain.

   (d) Let $f : X \to X$ be defined by $f(x, y) = (|x|, y)$. What is the induce homomorphism f_* on the fundamental group of X?

   (e) Let $f : X \to X$ be defined by $f(x, y) = (x, |y|)$. What is the induce homomorphism f_* on the fundamental group of X?

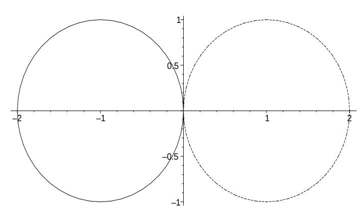

Figure 1:

3. Let $\pi : X \to Y$ be an identification map of path connected topological spaces. Let p be a base point in X and let $q = \pi(p)$ be a base point in Y.

   (a) What would it mean for π to have the path lifting property for paths that start at q?

(b) What would it mean for π to have the homotopy lifting property?

(c) Suppose that $X = \mathbb{R}^2$ and $Y = T^2$ is obtained as the identification space for the equivalence relation $(x, y) \equiv (x + m, y + n)$, for $m, n \in \mathbb{Z}$ on $\mathbb{R}^2$. What is the lift of the diagonal loop in T^2?

(d) Suppose that $\alpha : [0, 1] \to T^2$ is homotopically trival but not constant. Describe its lift to $\mathbb{R}^2$.

4. Let K be a simplicial complex.

   (a) Define what it means for the realization $|K|$ to be a simplicial manifold of dimension n.

   (b) Define what it means for the realization $|K|$ to be a simplicial manifold with boundary of dimension n.

   (c) Prove the following: If $|K|$ is a simplicial manifold of dimension n and $|L|$ is the simplicial manifold of dimension n with boundary obtained by removing the interior of an n simplex from $|K|$, then $\chi(L) = \chi(K) + (-1)^{n+1}$.

5. Let K be a simplicial complex and let $|K|$ be the realization of K.

   (a) Define the cone on $|K|$, $C(|K|)$.

   (b) Construct a simplicial complex CK whose realization $|CK|$ is homeomorphic to $C(|K|)$.

   (c) Prove that $\chi(CK) = 1$. (*Hint:* How should you think about counting simplices of CK?)