Math 241

Quiz 1 Sample Solutions
January 25, 2010

Show all your calculations.

1. Figure 1 shows a set $\mathcal{S}$ in the plane. Express the $\mathcal{S}$ in set notation.

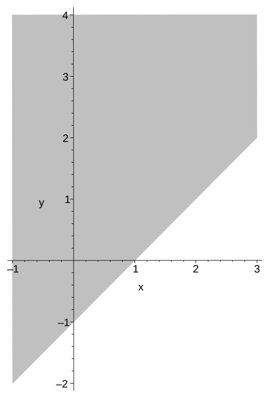

Figure 1:

$$\mathcal{S} = \{(x, y) : \ -1 \leq x \leq 3 \text{ and } x - 1 \leq y \leq 4\}$$

2. Let $\mathbf{u} = (u_1, u_2, u_3)$, $\mathbf{v} = (v_1, v_2, v_3)$, and $\mathbf{w} = (w_1, w_2, w_3)$ be vectors in $\mathbb{R}^3$ and let $a \in \mathbb{R}$. Prove that the dot product satisfies $(\mathbf{u} + a\mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + a(\mathbf{v} \cdot \mathbf{w})$.

   Begin on the left side, expand using the coordinate expressions for the vectors, use the algebraic properties of operations involving real numbers, then reassemble to obtain the right side. The following carries out these steps:

$$\begin{aligned}
(\mathbf{u} + a\mathbf{v}) \cdot \mathbf{w} &= ((u_1, u_2, u_3) + a(v_1, v_2, v_3)) \cdot (w_1, w_2, w_3) \\
&= (u_1 + av_1, u_2 + av_2, u_3 + av_3) \cdot (w_1, w_2, w_3) \\
&= (u_1 + av_1)w_1 + (u_2 + av_2)w_2 + (u_3 + av_3)w_3 \\
&= u_1w_1 + av_1w_1 + u_2w_2 + av_2w_2 + u_3w_3 + av_3w_3 \\
&= (u_1w_1 + u_2w_2 + u_3w_3) + a(v_1w_1 + v_2w_2 + v_3w_3) \\
&= \mathbf{u} \cdot \mathbf{w} + a(\mathbf{v} \cdot \mathbf{w}).
\end{aligned}$$

3. In $\mathbf{R}^3$, let $\mathbf{v} = (1, -1, 1)$ and $\mathbf{w} = (2, 3, 0)$.

(a) Compute $P_{\mathbf{w}}(\mathbf{v})$.

Use the formula $P_{\mathbf{w}}(\mathbf{v}) = \frac{\mathbf{w}\cdot\mathbf{v}}{\mathbf{w}\cdot\mathbf{w}}\mathbf{w}$.

$$P_{\mathbf{w}}(\mathbf{v}) = \frac{(1,-1,1)\cdot(2,3,0)}{(2,3,0)\cdot(2,3,0)}(2,3,0) = \frac{-1}{13}(2,3,0) = (-\frac{2}{13}, -\frac{3}{13}, 0).$$

(b) Find a vector perpendicular to both $\mathbf{v}$ and $\mathbf{w}$.

The cross product of $\mathbf{v}$ and $\mathbf{w}$ is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$.

$$\mathbf{v} \times \mathbf{w} = (1,-1,1) \times (2,3,0) = (-3, 2, 5).$$