Math 134

Final Exam Sample

May 5, 2011

You may use your calculator and integral tables. Show all your work in the blue book.

1. (15 pts.) Let F be defined by the integral:

$$F(x) = \int_0^x \frac{1}{1+t^4}\,dt$$

   (a) What is $F'(x)$?

   (b) Which of the following is correct, explain:
   - F is always increasing.
   - F is always decreasing.
   - F is increasing for some values of x and decreasing for others.

   (c) Find the interval(s) on which F is concave up and the interval(s) on which F is concave down.

2. (15 pts.) Let $f(x) = \sin(\pi x^2)$. Let L_4 denote the left endpoint approximation to $f(x)$ on the interval $[a, b] = [0, 1]$ that uses 4 rectangles.

   (a) Sketch the rectangles for L_4 on a graph of f on the interval $[0, 1]$.

   (b) Calculate L_4. Be sure to show the expanded form of your sum before evaluating the sum. Your final answer should be a number.

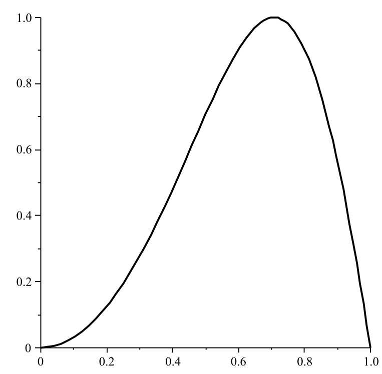

Figure 1:

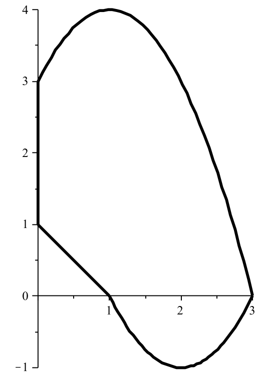

Figure 2:

3. (14 pts.)Let A be the region shown in Figure 2. It lies

   - above the segment $y = 1 - x$ and the curve $y = x^2 - 4x + 3$,
   - below the curve $y = 3 + 2x - x^2$, and
   - to the right of the y-axis.

   Find the area of A.

4. (36 pts.) Evaluate the following indefinite and definite integrals.

   (a)
   $$\int \frac{\sin(s)}{\cos^2(s)}\, ds$$

   (b)
   $$\int_1^e t^3 \ln(t)\, dt$$

   (c)
   $$\int \frac{3x^2 + x + 4}{(x-2)(x^2+2)}\, dx$$

5. (20 pts.) Let A be the region in the plane lying below the graph of $y = \sin(x)$ and above the x-axis for $0 \leq x \leq \pi$.

   (a) Sketch the region A.

   (b) Find the volume of the solid obtained by rotating A about the x-axis. (*Hint:* Which method should you use?)

   (c) Find the volume of the solid obtained by rotating A about the line $x = -2$. (*Hint:* Which method should you use?)

6. (20 pts.) (Hypothetical) The time to failure of a certain popular brand of notebook computer is modeled by an exponential density function with mean 4 years.

   (a) Write out the formula for this function.

   (b) If 400 Holy Cross first-year students purchase one of these computers when they arrive on campus in August, how many can expect to have working computers in August of their sophomore year?

   (c) Will half of these 400 computers fail by the time these students graduate (which is 45 months after they entered)?

7. (20 pts.) Consider the differential equation:

$$\frac{dP}{dt} = 2(5 - P).$$

   (a) Find the general solution to the equation.

   (b) Find the particular solution that satisfies $P(0) = 10$.

8. (20 pts.) Wallabies are marsupials (the young mature in an external pouch on the mother's stomach) related to kangaroos. There is a small colony of wallabies on the Isle of Man off the coast of England. (All true so far.) Consider the following logistic differential equation model for this population P.

$$\frac{dP}{dt} = .01 \cdot P(100 - P).$$

   (a) What is the carrying capacity for this model?

   (b) For which values of P will the population increase? For which values of P will the population decrease?

   (c) Sketch the line field for this model on the rectangle $0 \le t \le 100$ and $0 \le P < 150$. (*Hint:* It suffices to sketch the lines when $P = 25$, 50, 75, 100, and 125.)

   (d) Based on your plot, sketch a solution curve to the equation for the initial condition $P(0) = 25$.

9. (20 pts.)

   (a) Find the following limit:

$$\lim_{n\to\infty} \frac{n^2 + (-1)^n}{(-n)^3 + 1}$$

   (b) Find the sum of the following series:

$$\sum_{n=2}^{\infty} \frac{3^2}{4^n}$$

   (c) Does the following series converge or diverge? (*Hint:* What test should you use?)

$$\sum_{n=1}^{\infty} \frac{n + 1000}{n^3 + 1}$$

10. (20 pts.) Consider the series

$$\sum_{n=1}^{\infty} \frac{x^n}{4^n n^2}$$

(a) Find the radius of convergence of the series.

(b) Find the interval of convergence of the series.

(c) Find the derivative of this series.

11. (20 pts.)

(a) What is the MacLaurin series for the function $y = e^x$?

(b) Use your answer to (a) to write a series for $y = e^{-x}$.

(c) Use your answers to (a) and (b) to write a series for the function $C(x) = \frac{1}{2}(e^x + e^{-x})$.

(d) Based on the terms that appear in your series for $C(x)$, what sort of symmetry does $C(x)$ have? Explain.