3/18/20 Topology Review Part A: Simplicial Complexes and Filtrations

$$X = \{ x_1, \ldots, x_k, e_{12}, e_{13}, \ldots, f_{123}, \ldots, t_{2379} \ldots \}$$

or $$= \{ \sigma_1 \ldots \sigma_k, \sigma_{12}, \sigma_{13}, \sigma_{123}, \sigma_{2379}, \ldots \}$$

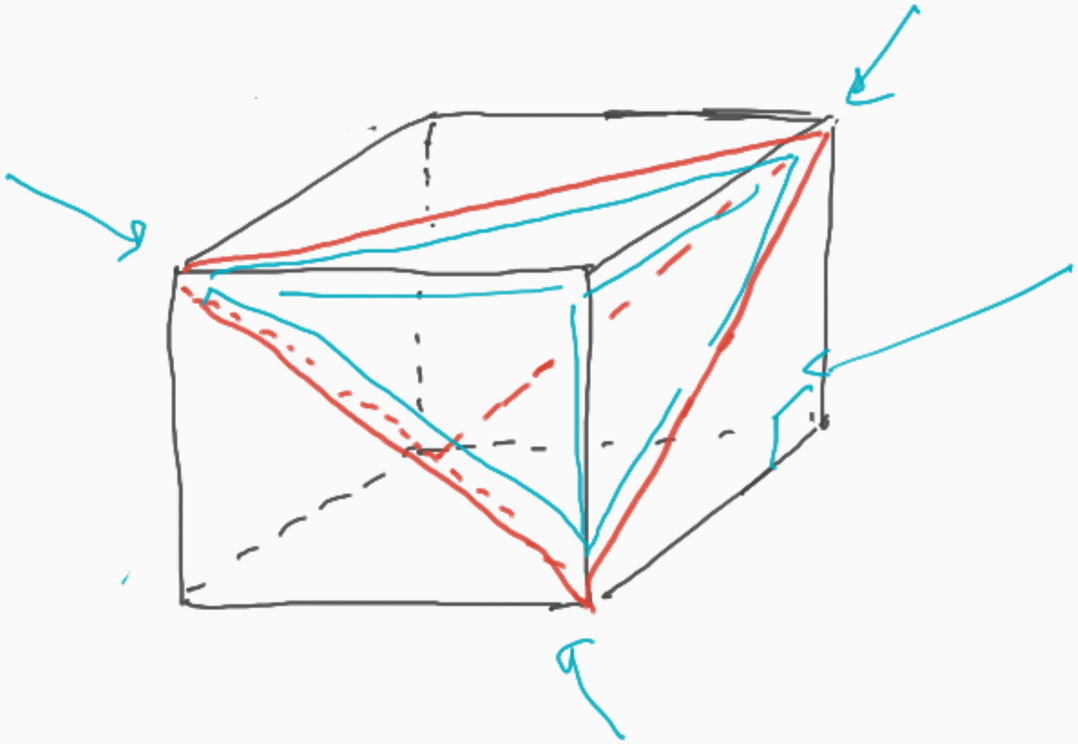
$\sigma_{i_0 i_1 \ldots i_k}$ = convex hull of $k+1$ pts in general position

Rules: ① $\sigma_{i_0 \ldots i_k} \in X \Rightarrow$ all faces also in X

e.g. $f_{125} \in X \Rightarrow x_1, x_2, x_5, e_{12}, e_{15}, e_{25} \in X$

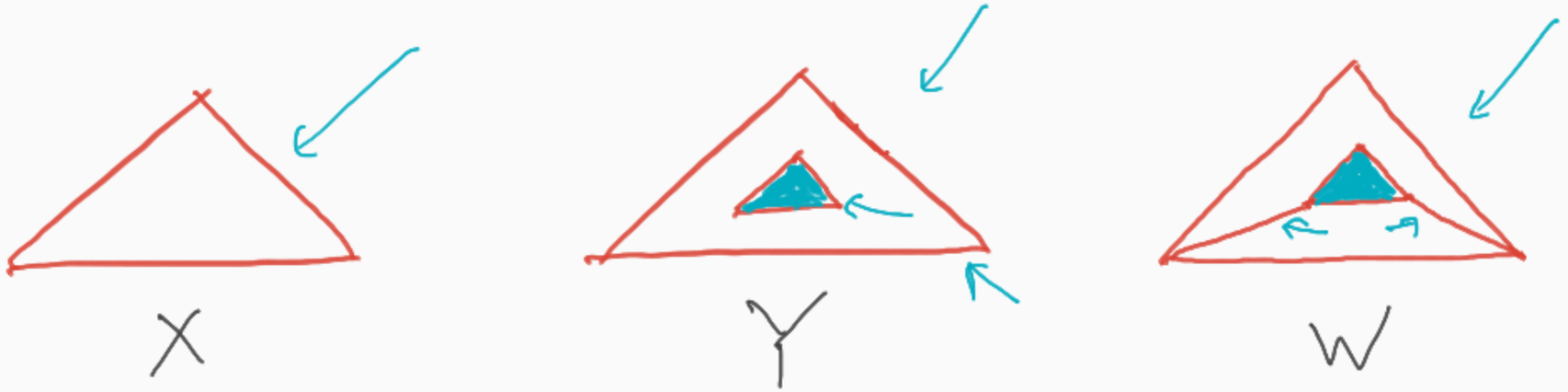
② $f_{123} \cap t_{1379} = e_{13} \in X$

Examples:



Lab 1: Cube with 5 tetrahedra

Handout 3:

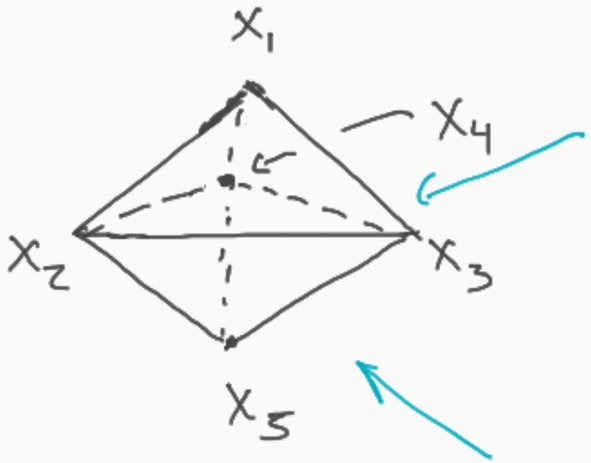


X : connected, 1 tunnel

Y : not connected, 1 tunnel

W : connected, 2 tunnels

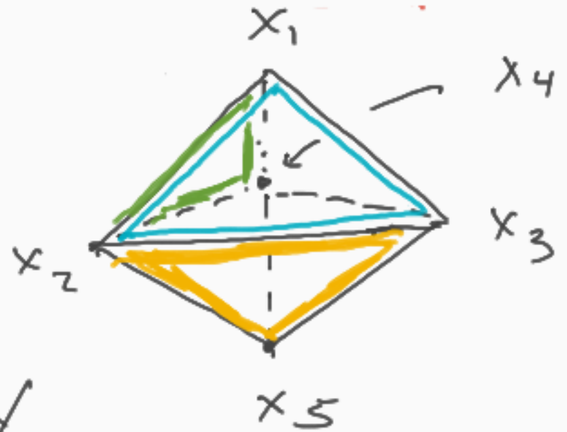
Homework 5:



X

- Hollow, 7 faces
- No tunnels

Two enclosed cavities or voids



Y

- Hollow 4 faces (f_{123}, f_{124}, f_{235} removed)
- 1 Tunnel

No voids

Filtrations

A way to construct a simplicial complex in steps.

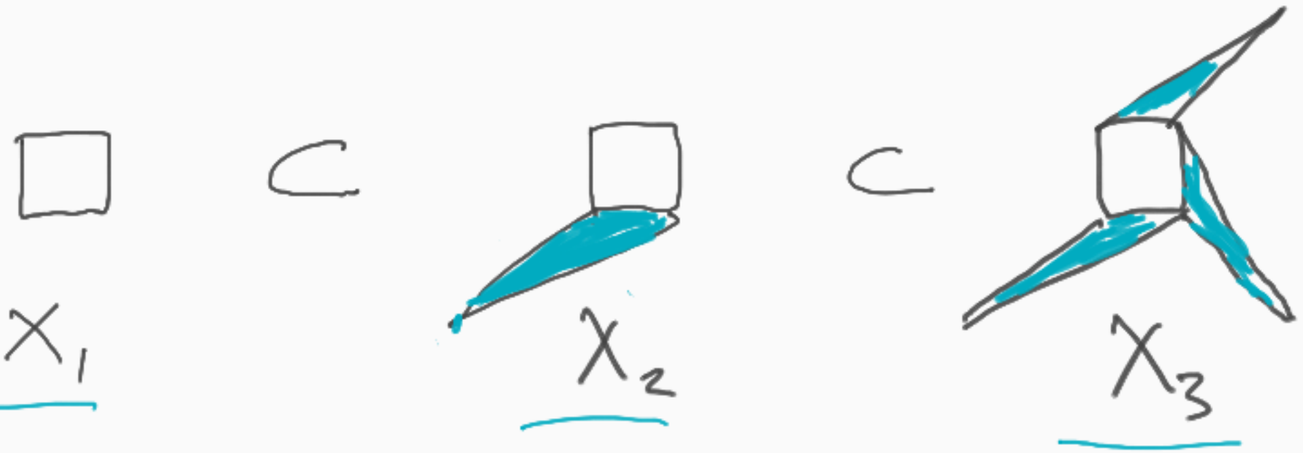
$$X_1 \subset X_2 \subset X_3 \subset \cdots \subset X_n = X$$

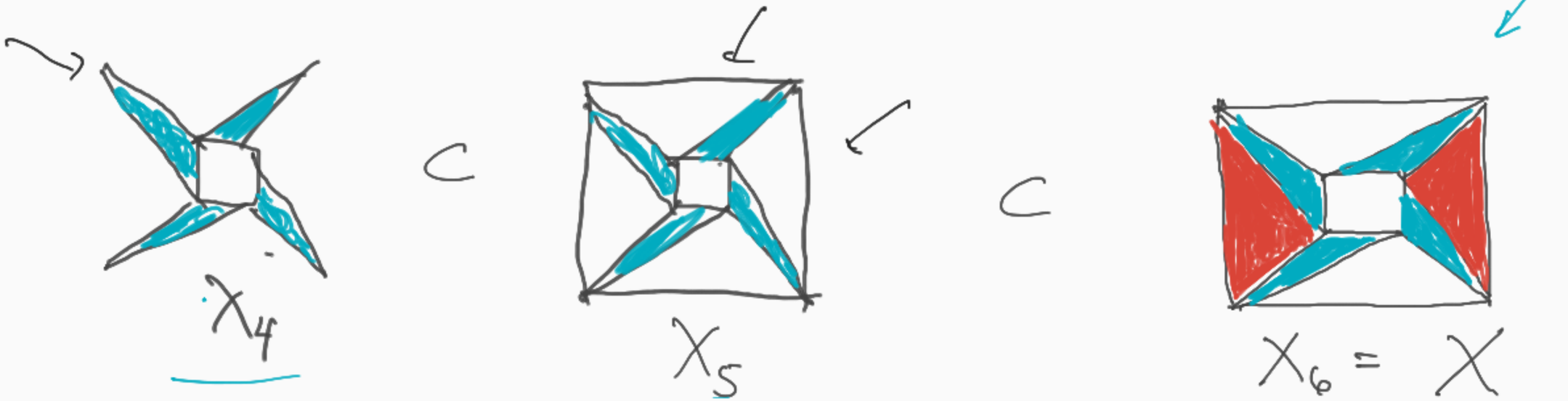
- Each X_i is a simplicial complex, a sub-complex of X.
- If $\sigma_{i_0 \ldots i_k} \in X_i$, then $\sigma_{i_0 \ldots i_k} \in X_{i+1}$
- Containment $\Leftrightarrow$ Inclusion function

$$In_i : X_i \to X_{i+1}$$

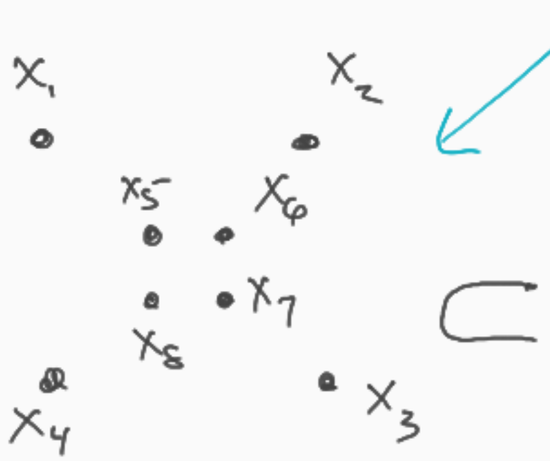
Examples:

A. By construction

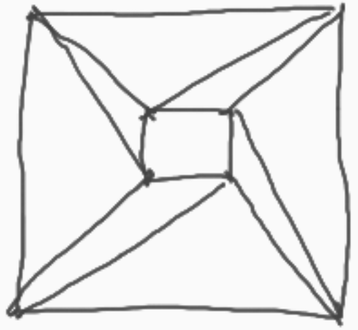




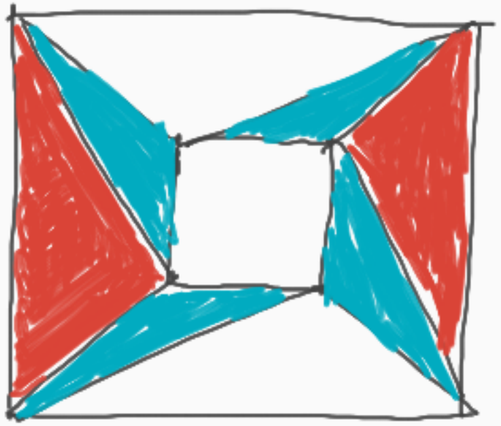
B. By skeleta



$\subset$



$\subset$



X_0 X_1 $X_2 = X$

$X_0 = \{x_1, \ldots, x_8\}$

$X_1 = \{\underbrace{x_{1,\ldots,7} x_8}_{X_0}, e_{12}, e_{23}, \ldots\}$

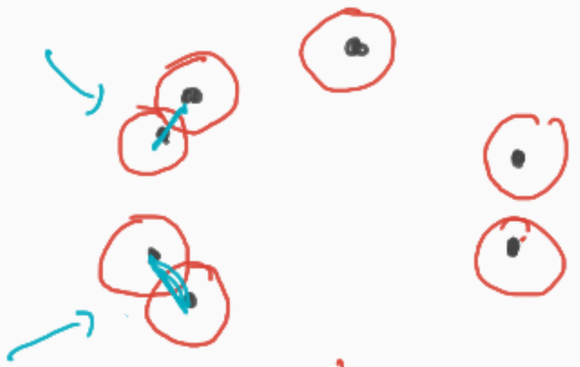
$X_2 = X_1 \cup \{f_{478}, \ldots\}$

C. By distance (Rips)

Points in $\mathbb{R}^2$

$d > 2r_1$

r_1 small



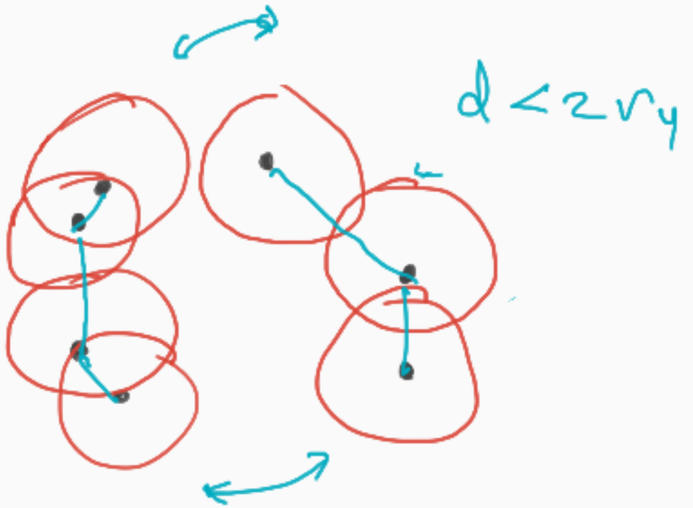
$d < r_2 + r_2$

$d < 2r_2$

r_2 larger

$d < 2r_3$

$r_3 > r_2$



$d < 2r_4$

$r_4 > r_3$