Practice Problems for Chapter 10

Write down the first 3 terms of the Taylor Series. Whenever possible, use a basic Taylor Series as a shortcut.

1. \( f(x) = \sin x \); \( a = \frac{\pi}{2} \)
2. \( f(x) = \sqrt{1+x} \); \( a = 1 \)
3. \( f(y) = \sqrt{1-y} \); \( a = 0 \)

By recognizing each series as a Taylor Series evaluated at a particular value of \( x \), find the sum of the convergent series.

4. \( 1 + \frac{2}{1!} + \frac{4}{2!} + \frac{8}{3!} + \ldots + \frac{2^n}{n!} + \ldots \)
5. \( 1 + \frac{1}{4} + (\frac{1}{4})^2 + (\frac{1}{4})^3 + \ldots + (\frac{1}{4})^n + \ldots \)
6. \( \frac{1}{2} - \frac{(\frac{1}{2})^2}{2} + \frac{(\frac{1}{2})^3}{3} - \frac{(\frac{1}{2})^4}{4} + \ldots + \frac{(-1)^n (\frac{1}{2})^{n+1}}{n+1} + \ldots \)

7. Consider the error in using the approximation \( \sin \theta \approx \theta \) on the interval \([-1,1]\). Estimate the magnitude of the largest possible error. First, explain why \( \theta \) is both \( P_1(\theta) \) and \( P_2(\theta) \). Use \( n=2 \) for your work.