Calculus Activity

This is worth 15 activity points.

Water Tank Problem
From P. Taylor Calculus: The Analysis of Functions

At time $t = 0$, water begins to flow from a hose into an empty tank at the rate of 40 liters per minute. This flow rate is held constant for two minutes, at which time the tank has 80 liters. At that time ($t = 2$), the water pressure is gradually reduced, until at time $t = 4$, the flow rate is 5 liters/min. This flow rate is held constant for the final two minute period, at which time ($t = 6$), the tank contains 120 liters.

A. Draw a graph of the volume $V$ (liters) of water in the tank against the time $t$ (minutes) for the time interval $[0,6]$.

B. What is the average rate of flow into the tank over the entire six minute period?

C. Now, suppose that, in addition to the above sequence of events, a pump is started up at time $t = 2$, and for the next four minutes, pumps water out of the tank at the constant rate of 15 liters/min. What we have called $V$ above now represents the total amount of water that has flowed INTO the tank at time $t$. Let $W$ represent the total amount of water pumped OUT of the tank at time $t$. Plot $W$ on the same set of axes. It should be an increasing function.

D. If you want to know the volume of water in the tank at any one time, explain in a sentence or two how you can use the separate graphs of $V$ and $W$ to interpret this quantity.

E. Explain in a few sentences how you can use your graph to estimate the time when the water level in the tank is at a maximum. Explain.

F. What is the instantaneous flow rate from the hose into the tank at this time (when the water level is at a maximum)?

G. If we continue the action past time $t = 6$, with the constant flow into the tank of five liters per minute and the constant flow out of the tank of 15 liters per minute, at what time will the tank be empty? Show how to interpret this graphically using the separate graphs of $V$ and $W$.

H. Sketch a new graph that shows the total amount of water in the tank, $T = V - W$, at any time $t$. 