Properties of exponents

\[ a^{x+y} = a^x a^y \]
\[ a^{x-y} = a^x a^{-y} = a^x/a^y \]
\[ (a^x)^y = a^{xy} \]
\[ (ab)^x = a^x b^x \]

Example: Bacteria experiment, \( N(t) = N_0 a^t \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>( N(t) )</th>
<th>Test for exponential function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>( \Delta t ) constant</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>( \Delta N ) has constant ratios from subsequent points.</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

Here: \( \frac{10}{5} = 2 \); \( \frac{20}{10} = 2 \); \( \frac{40}{20} = 2 \)

Note: In formula \( N(t) = N_0 a^t \), \( N_0 \) is value when \( t = 0 \) and if the \( t \)-values are spaced by one, then \( a \) is the constant ratio.

Here, \( N(t) = 5 \cdot 2^{t/10} \).

Found by plugging in two points into model equation \( N(t) = N_0 a^t \) & determining value for \( a \).
Forms of exp'1 eqns

\[ P(t) = P_0 \cdot a^t \]

- \( a > 1 \) growth factor
- \( 0 < a < 1 \) decay factor

Ex: \( a = 1.026 \)
means growth of 2.6%.

Ex: \( a = 0.6 \)
means decay of 40%.

\[ P(t) = P_0 \cdot (1 \pm r)^t \]

where \( r \) is the percentage of
the growth or decay rate...
add if growth (so \( a > 1 \)) & subtract if decay.

\[ P(t) = P_0 \cdot e^{kt}, \ k > 0 \] growth
\[ P(t) = P_0 \cdot e^{-kt}, \ k > 0 \] decay

\( k \) is the "continuous" rate of
growth or decay.
Ex (1.2#36)

in early 1960's, strontium-90 was released during nuclear tests & got into peoples bones. If the \( \frac{1}{2} \)-life is 29 years, what fraction of strontium-90 absorbed in 1960 remained in bones in 2000?

Ex (1.2#37)

Write \( p = 7e^{-\frac{\pi}{t}} \) in other forms.

Ex (1.2#19)

Give a possible formula.

\[ y \\
(\text{-}1.8) \quad (0,2) \]

Ex (1.2#9)

A town with a population of 1000 requests a model to approx. its population when
(a) it increases by 50 people/yr.
(b) it incr. by 5\% /year.