

Mathematics 132 – Calculus for Physical and Life Sciences 2
Exam 3 – Review Sheet
April 16, 2009

General Information

As announced in the course syllabus, the third midterm exam of the semester will be given at 6:00pm on Thursday, April 23. The exam will take place in O’Neil 112. The format will be similar to that of the first two midterms and the exams from last semester.

- **Bring a photo ID to the exam.**
- The exam will be designed to take an hour but you will have an extra 30 minutes to work and check your solutions.
- You will be given a TI-30 scientific calculator for the exam which does NOT have graphing capabilities so be prepared to answer questions without your personal calculator. (Note: Some of you may have one of these calculators purchased for use in Chemistry courses here. That is also OK.)
- Use of cell phones, I-pods, and all other electronic devices *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are turned off).

What will be covered

The exam will cover the material since the last exam (Problem Sets 7, 8, 9 and 10), namely the following material from sections 7.2 - 7.5, and 8.1 - 8.5 of Stewart:

1. Differential equations (Chapter 7)
 - (a) Euler’s method for approximating solutions,
 - (b) separable equations and solution by separation of variables and integration,
 - (c) exponential growth and decay problems; other growth and decay problems (be prepared to set up a differential equation matching a verbal description, solve it, and use the solution to answer questions),
 - (d) logistic equations – know the form

$$y' = ky \left(1 - \frac{y}{M}\right)$$

of the logistic equation, the properties of the solutions, and the analytic formula

$$y = \frac{M}{1 + be^{-kt}}$$

for the solutions.

2. Sequences and Series (Chapter 8)

- (a) the concepts of sequences and series (and the difference between them!), the concepts of convergence for sequences and for series (and the difference between them!)
- (b) key classes of examples such as geometric series, the harmonic series, p -series, etc.
- (c) general convergence tests: the Integral Test, the Alternating Series Test, the Test for Divergence, the Ratio Test for absolute convergence (in addition to using these on specific examples, be prepared to give careful *statements* of the theorems concerning each test – what hypotheses are necessary, what conditions imply convergence or divergence, and the conclusion in each case).
- (d) power series, radius of convergence, interval of convergence

Important Note: Some of the problems on this exam (in particular solving separable differential equations and applying the Integral Test for series) will require you to set up and compute integrals to find the quantity that is asked for. In addition to knowing how to set up the required integral, *any* of the methods of integration tested on the first exam (i.e. basic rules, u -substitution, integration by parts, trigonometric substitution, partial fractions, or consultation of a table of integrals) might be required to evaluate the integral. Again, there is a high degree of cumulativeness built into this material.

There will be a review for the exam in class on Wednesday, April 22.

You should review all examples presented in class and the homework problems from the relevant assignments.

Review Problems

- Chapter 7 Review: 3b, 5, 7, 11
- Section 7.2: 19a
- Section 7.3: 1, 7, 9, 11, 33, 37
- Section 7.4: 9, 11, 13, 17
- Section 7.5: 1, 7
- Section 8.1: 7, 9, 13, 15, 19, 41
- Section 8.2: 11, 15, 19, 25
- Section 8.3: 21 (use the integral test)
- Section 8.4: 19, 23
- Section 8.5: 7, 9, 11
- Chapter 8 review: 1, 3, 5, 9, 11, 13, 15, 19, 31, 33

Sample Exam Questions

I. All parts of this question refer to the differential equation

$$y' = y(4 - y)$$

- (1) Use Euler's method to approximate the solution of this equation with $y(0) = 1$ for $0 \leq x \leq 1$ using $n = 4$.
- (2) This is a separable equation, find the general solution and determine the constant of integration from the initial condition $y(0) = 1$.

II. Find the general solutions of the following differential equations

(1) $y' = \frac{y}{x(x+1)}$

(2) $y' = \frac{\sqrt{1-x^2}}{e^{2y}}$.

III. Cesium 137 decays at a rate proportional to the amount of itself present and has a half-life of 30 years. Suppose we have a sample of 100 mg. of Cesium 137.

- (1) Write a differential equation whose solution is $y(t)$, the amount of Cesium 137 left after t years.
- (2) Give a formula for the amount $y(t)$ of Cesium 137 left after t years.
- (3) After 98 years, how much Cesium 137 is left of the sample?
- (4) How long will it take until only 1 mg of the sample is left?

IV.

- (1) Does the sequence $a_n = n \ln(1 + n)$ converge? Why or why not? Does the infinite series $\sum_{n=1}^{\infty} n \ln(1 + n)$ converge? Why or why not?
- (2) Use the Integral Test to determine whether or not

$$\sum_{k=1}^{\infty} \frac{k}{e^k}$$

converges.

- (3) Use the Ratio Test to determine whether or not

$$\sum_{k=0}^{\infty} \frac{3^n}{n!}$$

converges.

(4) Determine (with justification!) whether or not the following series converge:

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}, \quad \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{\pi^{2n}}, \quad \sum_{n=1}^{\infty} \frac{1}{n^{1.01}}.$$

V. For each of the given power series, find the interval of convergence.

$$f(x) = \sum_{n=1}^{\infty} \frac{(2x)^n}{\sqrt{n}}, \quad g(x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-5)^n}{n \cdot 3^n}.$$

(In particular, give the radius of convergence, and investigate convergence at the endpoints.)