

General Information

As announced in the course syllabus, the second midterm exam of the semester will be given at 6:00 p.m. on Thursday, March 19. The format will be similar to that of the first midterm and the exams from last semester.

• **Bring a photo ID to the exam.**

• The exam will be designed to take an hour but you will have an extra 30 minutes to work and check your solutions.

• You will be given a TI-30 scientific calculator for the exam which does NOT have graphing capabilities so be prepared to answer questions without your personal calculator. (Note: Some of you may have one of these calculators purchased for use in Chemistry courses here. That is also OK.)

• Use of cell phones, I-pods, and all other electronic devices *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are turned off).

What will be covered

The exam will cover the material since the last exam (Problem Sets 5 and 6), namely the following material from sections 6.1, 6.2, 6.3, 6.4, 6.5, 7.1, 7.2 of Stewart:

1. Areas between curves
2. Volumes of solids with known cross-sections and solids of revolution
3. Arc length of curves
4. Average value of $f(x)$ on an interval $[a, b]$
5. Center of mass of a thin rod with given mass density. Center of mass of a thin plate with constant mass density.
6. Modeling with differential equations. Verifying that a given function or family of functions is a solution to a given differential equation. Direction fields

Important Note: Most of the problems on this exam will require you to set up and compute an integral to find the quantity that is asked for. In addition to knowing how to set up the required integral *any* of the methods of integration tested on the first exam (i.e. basic rules, u -substitution, integration by parts, trigonometric substitution, partial fractions, or consultation of a table of integrals) might be required to evaluate the integral. In other words, this exam is *really a cumulative exam on the first two-thirds of the semester*. Especially if you did poorly on the first exam, you will

need to begin your review for this exam by going back and looking at the material from sections 5.1 through 5.8 in Stewart.

There will be a review for the exam in class on Wednesday, March 18.

Review Problems

Section 6.1: #1, 3, 5, 9, 11, 29 (place the origin at the center of the large circle, and note that the endpoints of the arc on the smaller circle are at opposite ends of a diameter of that circle), 37

Section 6.2: # 3, 5, 7, 13, 21 (and evaluate),

Section 6.3: #5, 7, 9 (look for an algebraic simplification to integrate), 17, 19 (use the table for these)

Section 6.4: #1, 5, 13

Section 6.5: #35, 37

Review of Chapter 6: #1, 7, 11, 15, 17

Section 7.1: #1, 4 (answer $k = 5/2$ or $-5/2$), 11,

Section 7.2: #1, 7

Sample Exam Questions

This list is longer than the actual exam will be (to give you some idea of the range of different questions that might be asked). Unless otherwise directed, you may use any entry of the Table of Integrals from the text that applies. Please note that the numbering starts at III. (This is not a mistake.)

III.

(A) Let R be the region in the plane bounded by $y = 3 - x^2$ and the x -axis.

1. Sketch the region R .
2. Find the area of R .
3. Find the volume of the solid generated by rotating R about the x -axis.

(B) Let R be the region in the plane bounded by $y = 3x$ and $y = x^2$.

1. Sketch the region R .
2. Find the area of R .
3. Find the volume of the solid generated by rotating R about the x -axis.
4. Find the volume of the solid generated by rotating R about the y -axis.

(C) Let R be the region in the plane bounded by $y = \cos(\pi x)$, $y = 1/2$, $x = -1/3$ and $x = 1/3$.

1. Sketch the region R .
2. Find the area of R .
3. Find the volume of the solid generated by rotating R about the x -axis.

IV. The height of a monument is 20m. The horizontal cross-section of the monument at x meters from the top is an isosceles right triangle with legs $x/4$ meters. Find the volume of the monument.

V.

(A) Set up and evaluate the integral to compute the arclength of the curve $x = 3t^2$, $y = 2t^3$, $0 \leq t \leq 2$.

(B) Set up and evaluate the integral to compute the arclength of the curve $y = \frac{1}{6}(x^2 + 4)^{3/2}$, $0 \leq x \leq 3$. (Hint: the arclength integral simplifies to a manageable form if you are careful with the algebra.)

VI.

(A) Find the average value of $f(x) = \sqrt{1-x^2}$ on the interval $[0, 1/2]$. (Use trigonometric substitution, not the table.)

(B) Find the average value of $f(x) = x\sqrt{1+x^4}$ on the interval $[0, 2]$.

VII.

(A) Find the center of mass of a thin rod 7 yards long if the mass density x yards from the left end of the rod equals $\rho(x) = x \ln(x+1)$ Kg/yd.

(B) Find the center of mass of a this plate in the shape of the region bounded by the curves $f(x) = e^x$, $x = 0$, $x = 1$ and $y = 0$

VIII. Consider the differential equation $y' = x^2(y+1)$.

(A) Verify that every member of the family of functions

$$y = Ce^{\left(\frac{x^3}{3}\right)} - 1$$

is a solution for the differential equation above.

(B) Find the solution to differential equation above which also satisfies $y(0) = -2$.

IX. Match the following differential equations with the slope field (direction field) below. Note that there are only 3 plots, so the correct answer for one equation is "None". Circle the correct answers.

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|--------------------|-----|------|-------|------|
| (1) $y' = x$ | (I) | (II) | (III) | None |
| (2) $y' = 1 + y^2$ | (I) | (II) | (III) | None |
| (3) $y' = 1 - y$ | (I) | (II) | (III) | None |
| (4) $y' = x - y$ | (I) | (II) | (III) | None |

