Mathematics 132 – Calculus for Physical and Life Sciences 2
Exam 1 – Review Sheet
Spring 2009

General Information

The first midterm exam take place on Thursday, February 19, at 6:00 p.m. in O'Neill 112. You will have 90 minutes to work on the exam if you need that much time. The format will be similar to that of the midterm exams last semester. The exam will cover the material since the start of the semester, including the following material from sections 5.1 through 5.8 and 5.10 of Stewart:

1. Riemann sums and the definition of the definite integral.
2. The Fundamental Theorem of Calculus: Part 1: If $f(t)$ is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)\ dt$, then $F'(x) = f(x)$. Part 2: (the Evaluation Theorem) If $F(x)$ is an antiderivative of a continuous function $f(x)$ on $[a, b]$, then
$$\int_a^b f(x)\ dx = F(b) - F(a)$$
3. Antiderivatives graphically and numerically
4. Basic antiderivative rules: All rules coming from basic derivative formulas: Know $\int x^n\ dx$, $\int a^x\ dx$, $\int \sin(x)\ dx$, $\int \cos(x)\ dx$, $\int \frac{1}{x^2+1}\ dx$, $\int \frac{1}{\sqrt{1-x^2}}\ dx$, and so forth, plus the sum, and constant multiple rules
5. Integrals by substitution
6. Integrals by parts: Selecting appropriate u and v', computing u' and v, using the parts formula $\int u\ v'\ dx = uv - \int v\ u'\ dx$, then finishing the integral on the right. Recall that this might involve using parts again, or another method such as substitution.
7. Integration by trigonometric substitution.
8. Integration of rational functions by partial fractions (see summary distributed in class).
9. Integrals by the table: Recognizing the appropriate entry, using reduction formulas repeatedly if necessary. Be aware that a preliminary substitution might be necessary to take an integral to one of the forms in the table.
10. Improper integrals

Note: Some problems may ask you to carry out a particular integration method on a problem. Others may leave the choice up to you. *Be prepared for both types of questions!* **There will be a review for the exam in class on Wednesday, February 18.**

Review Problems

The Review Problems 1 - 46 and 55 - 61 at the end of Chapter 5 are good for preparation for this exam. It's not necessary to work all of them. But you should try a good selection and practice choosing a method at least for most of the integrals in problems 9 - 46.

Sample Exam Questions

I. Let

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 3 \\ x - 2 & \text{if } 3 \le x \le 5 \\ 13 - 2x & \text{if } 5 \le x \le 8 \end{cases}$$

A) Sketch the graph $y = f(x)$.

In the rest of the parts, $F(x) = \int_0^x f(t)\, dt$, where f is the function from part A.

B) Assuming $F(0) = 0$, Compute $F(1), F(2), F(3), F(4), F(5), F(6), F(7), F(8)$ given the information in the graph of f.

C) Are there any critical points of F? If so, find them and say whether they are local maxima, local minima, or neither. If not, say why not.

D) Sketch the graph $y = F(x)$ if $F(0) = 0$, and also if $F(0) = 2$.

II. Find the derivatives of the following functions

A) $f(x) = \int_0^x \sin(t)/t\, dt$.

B) $g(x) = \int_5^{x^3} \tan^4(t)\, dt$.

C) $h(x) = \int_{-3x}^{5x} e^{t^2} \sin(t)\, dt$.

III.

A) Compute $\int 5x^4 - 3\sqrt{x} + e^x + \frac{2}{x}\, dx$

B) Apply a u-substitution to compute $\int x(4x^2 - 3)^{3/5}\, dx$

C) Apply a u-substitution to compute $\int_1^2 e^{\sin(\pi x)} \cos(\pi x)\, dx$

D) Do you need partial fractions to compute

$$\int \frac{t^2 + 1}{t^3 + 3t + 3}\, dt?$$

Explain, and give a simpler method.

E) Apply integration by parts to compute $\int x^2 e^{-2x}\, dx$

F) Apply partial fraction decomposition to compute

$$\int \frac{1}{x(x-1)(x+2)}\,dx$$

G) Which trigonometric substitution would you apply to compute $\int \frac{1}{u\sqrt{a^2-u^2}}\,du$? What trigonometric integral do you get after making the substitution? Complete the derivation of the integral.

IV. Compute each of the integrals below using some combination of basic rules, substitution, integration by parts, the table of integrals, partial fractions, and trigonometric substition. You must show all work for full credit.

A)

$$\int \frac{x^3}{x^2+4x}\,dx$$

B)

$$\int (x^2+2x)^{3/2}\,dx$$

C)

$$\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx$$

D)

$$\int \frac{dz}{(z+3)^2\sqrt{z^2+6z+10}}$$

V. For each of the following integrals, decide if the integral converges or diverges. If the integral converges, evaluate it.

A)

$$\int_1^\infty \frac{\ln x}{x}\,dx$$

B)

$$\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx$$