College of the Holy Cross, Fall Semester, 2005
Math 131, Practice Final

1. Circle the number corresponding to the graph of each function. Each square in the figures is 1 unit by 1 unit, and the bold lines are the axes.

(a) $\frac{1}{2}x - 2$ I II III IV V VI VII VIII IX

(b) $\sin(4x)$ I II III IV V VI VII VIII IX

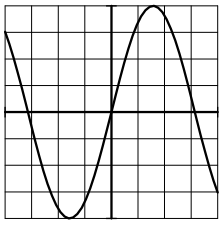
(c) $x^3 - 2x - 1$ I II III IV V VI VII VIII IX

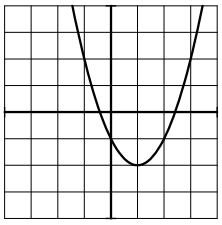
(d) $\dfrac{x}{x-1}$ I II III IV V VI VII VIII IX

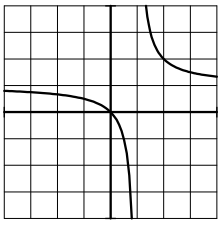
(e) e^{-x} I II III IV V VI VII VIII IX

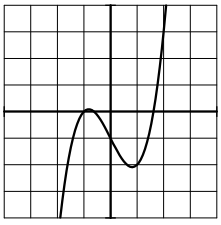
(f) $x^2 - 2x - 1$ I II III IV V VI VII VIII IX

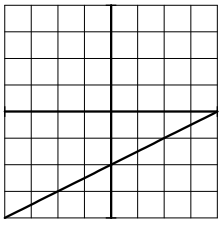
(g) $4\sin(x)$ I II III IV V VI VII VIII IX

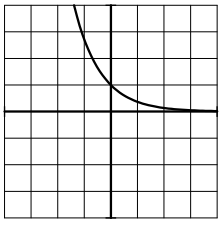
(I) 

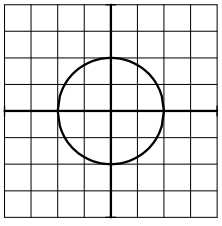
(II) 

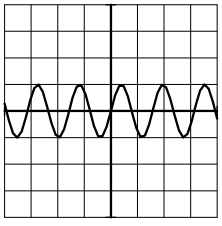
(III) 

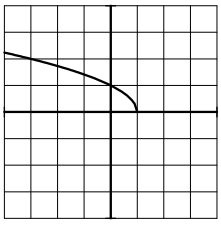
(IV) 

(V) 

(VI) 

(VII) 

(VIII) 

(IX) 

2. Compute the following limits.

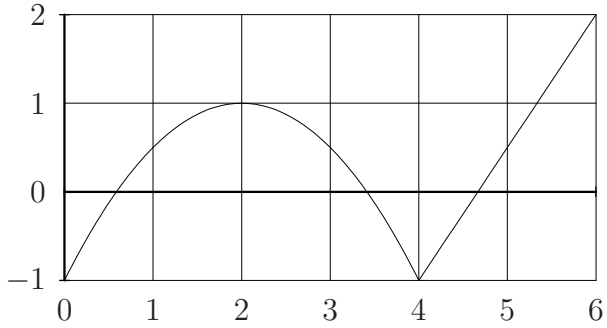
   (a) $\lim_{x \to 0} \dfrac{x^2 - 9}{3 - x}$

   (b) $\lim_{x \to 1} \dfrac{2^x - 2}{\ln(x)}$

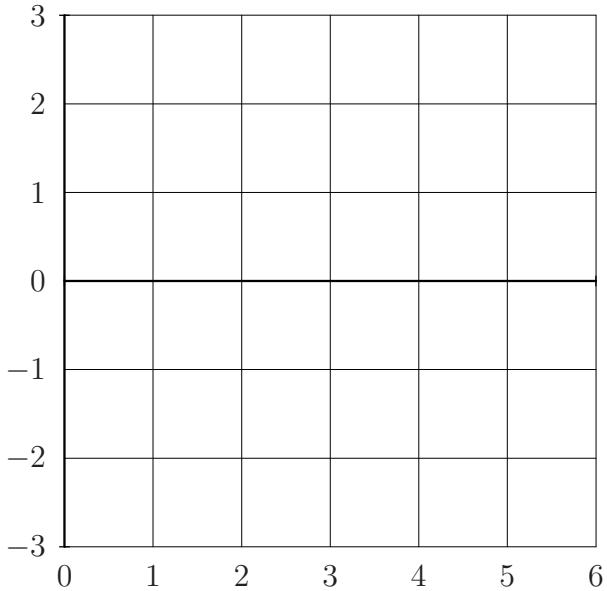
   (c) $\lim_{x \to \infty} \dfrac{4x^2 + 3e^{-x}}{5x^2 + 7e^{-x}}$

3. The graph of the function $f(x)$ is shown below. On the axes provided, sketch the graph of $f'(x)$. Be sure to label the points where f' is zero or undefined.

   Graph of f:



   Graph of f':



4. (a) Suppose $f(2) = 3$ and $f'(2) = -5$. Use the linear approximation of f at $x = 2$ to estimate the value of $f(1.97)$.

   (b) Suppose as in part (a) that $f(2) = 3$ and $f'(2) = -5$. Also suppose $g(3) = -1$ and $g'(3) = 7$. Let $h(x) = g(f(x))$. Find $h'(2)$.

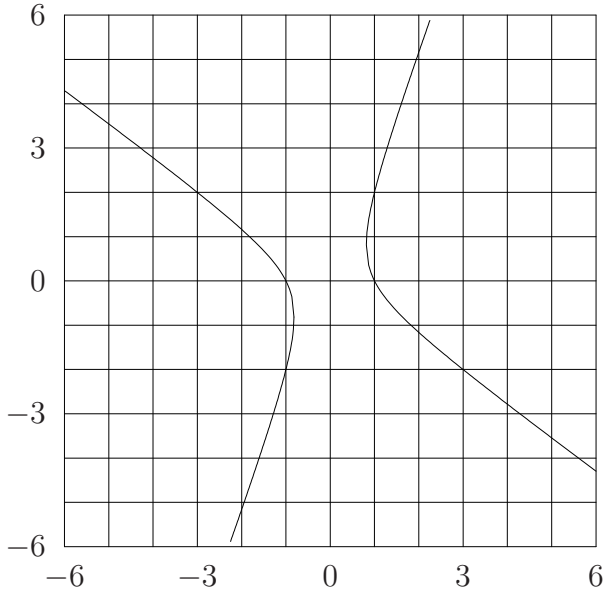
5. Compute the derivative of each function.

   (a) $f(x) = \dfrac{x^3}{2x^2+1}$

   (b) $g(x) = x^2\cos(3x)$

   (c) $h(x) = \ln(1+e^{\sqrt{x}})$

6. The hyperbola $2x^2 + 2xy - y^2 = 2$ is shown below.



   (a) Compute $\dfrac{dy}{dx}$ in terms of x and y.

   (b) Find the equation of the tangent line to the curve at the point $(1, 2)$ and sketch the tangent line on the figure above.

7. Suppose the position of an object is described by the parametric curve $x = t^3 - t^2$, $y = 3t^3 - 4t$, where t is measured in seconds.

   (a) Find the location of the object at time 2 seconds. Find the equation of the tangent line to the curve at this point.

   (b) At what time is the speed of the object zero?

8. Let $f(x) = x^3e^x$.

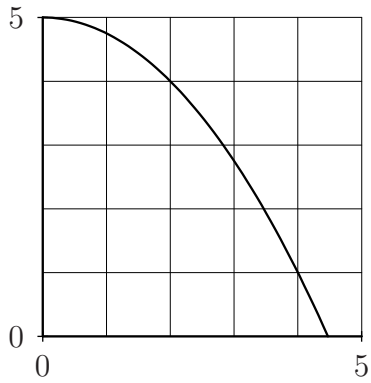
   (a) Find and classify (local min/max, or neither) the critical points of f.

   (b) Find the inflection points of f.

9. You have \$3 with which to construct a box with a square base. The material for the top and bottom costs 2 cents per square inch, while the material for the sides costs 1 cent per square inch. What dimensions maximize the volume of the box? *Hint:* First write the cost of the box and the volume of the box in terms of the dimensions of the box.

10. Suppose you are making a snow-person, and you begin by rolling a snowball in the snow. Assume the snowball always remains in the shape of a sphere. If the radius of the snowball increases at a constant rate of 2 inches per second, how fast is the volume of the snowball increasing when the radius is 6 inches? Give the units of your answer. The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

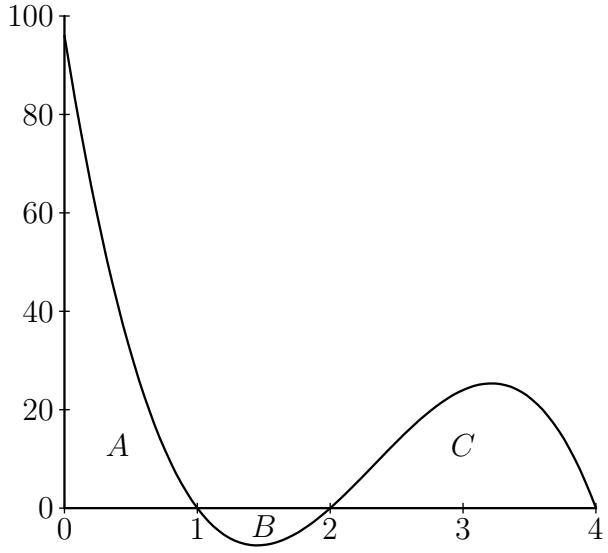
11. The graph of $f(x) = 5 - \frac{1}{4}x^2$ is shown below.



(a) Compute the left and right hand sums for f over the interval $1 \leq x \leq 4$ using $n = 3$ subintervals. On the figure above sketch the rectangles which represent the terms in the left and right hand sums.

(b) Which sum is a better estimate of the exact area under the graph of f over $1 \leq x \leq 4$? Explain.

12. The graph of $f(x)$ is shown below. The areas of the regions shown are $A = 37$, $B = 5$ and $C = 32$.



(a) Evaluate $\displaystyle\int_0^4 f(x)\,dx$

(b) Find the average value of f over the interval $1 \leq x \leq 4$.

13. Suppose the rate of growth of a population of bats living under a bridge, in thousands of bats per year, is

$$f(t) = 2 + \frac{t}{3}$$

where t is measured in years since 1990. Suppose there were 40 thousand bats in 1990.

(a) Write out an integral that represents the change in the bat population from 1990 to 2005.

(b) Compute the integral exactly (use the graph of f) and determine what the population will be in 2005.