Math 136, Lab #2,
Wednesday, October 8

The lab report is due Wednesday, October 15 in class. Work in groups of two and turn in only one report per group.

In class exercises

1. Computing derivatives. Maple has a built-in function called **D** that can be used to compute derivatives. For a previously defined function f, the command

`D(f);`

will give the derivative of f with respect to its input variable, whatever that may be. Try defining some easy functions, whose derivative you already know, to make sure it works. In many cases it will be useful to name the derivative function. For example, you can do this by typing **d:=D(f);**. Then, if you want to plot the derivative of f, you can type

`plot{d};` or `plot(d(x), x=-10..10)`

to specify an x range. To get the second derivative, you can use `D(d);` or `D(D(f));`

You can also use the commands

`diff(f(x),x);    diff(f(x),x,x);`

for the derivative and the second derivative of f

When Maple can, it will try to supply you with an *exact form* of the result of the calculation you asked for. For example, if $f(x) = \sin(x)$, then `f(`$\frac{\pi}{4}$`);` will return $\sqrt{2}/2$. Try it. If you want to get a decimal approximation instead, you can use the command `evalf(`$f(\pi/4)$`);` Try it.

2. Rational solutions. In this exercise we start with the solution $(-2, 1)$ of the equation $y^2 = x^3 + 9$ and compute other rational solutions by the tangent line method explained in class.

a) We define the equation, and tell MAPLE to think of y as an implicit function of x:

```
eq:=y(x)^2 = x^3+9;
```

Take the derivative of the previous equation with respect to x, and call the result deq:

`deq := diff(eq, x);`

Solve the previous equation for y' =diff(y(x),x)):

`slope := solve(deq, diff(y(x), x));`

b) What we found in part a) is the slope of the curve at the point (x, y). Now we compute the tangent line at the point $(-2, 1)$. Since we want to iterate the process, we define $a = -2, b = 1$ and use them in the formula of the slope to find the numerical slope at the point $(-2, 1)$. The subs function tells maple to replace x by a and y by b in the formula for the slope.

`a := -2; b := 1; m := subs(x = a, y = b, slope);`

Notice that you get something slightly unexpected for m, due to the fact that y is an implicit function of x. To see it is really a number, you can type `evalf(m);` to evaluate it numerically.

Define the equation of the tangent line:

`l := x-> b + m * (x - a);`

Find where the tangent line intersects the curve again, by plugging in $y = l(x)$ in the initial equation and solve for x:

```
solve( {x^3+9 = l(x)^2}, {x} );
```

Notice that you get the repeated root that we knew already, $x = -2$, together with a new one, the x-coordinate of the second intersection point. Call that root $x1$:

$$\texttt{x1} := \texttt{write the second root here};$$

The corresponding y values is simply:

$$\texttt{y1} := \texttt{l(x1)};$$

To visualize what we just did, let's plot the curve and the tangent line on the same graph:

```
plot({sqrt(x^3+9),-sqrt(x^3+9),l(x)},x=-3..10);
```

Modify the x-range to catch the second intersection point on the graph.

c) Now repeat the process, replacing a and b at the beginning of part b) by the new solution $(x1, y1)$ (you can use copy and paste). If you want to check that the numbers you get are indeed roots, you can compute: `y1^ 2-x1^ 3-9;`

Find a few more roots this way. Note that the we get only rational numbers in this way (ratios of two integers), but the numerator and denominator get large really fast!

3. Derivatives and limits. Define the function `f:=x->x^ x`. The domain of this function is $x > 0$.

a) Graph this function on the interval $x = 0..1$. Estimate the limit $\lim_{x\to 0^+} x^x$, and estimate the value of x where the function has a minimum. To get a good approximation for the value of x at the minimum, graph on the same plot f and its derivative `g:=D(f);` :

$$\texttt{plot(f(x), g(x), x = 0..1);}$$

and change the x range until you can get at least two decimal digits for the minimum.

b) What is $\lim_{x\to 0^+} f'(x)$? We will answer this question by computing the slope of secant lines starting at $x = 0$.

Plot the graph of f on intervals $0..h$ starting with $h = 0.1$:

$$\texttt{h := 0.1; plot(f(x), x = 0..h, scaling = constrained);}$$

Use the labels on the coordinate axes to estimate the slope of the graph. Repeat with smaller values for h. What seems to be the limit of the slopes? Prove your answer by using the formula for $f'(x)$ and calculus.

Homework (due Wed, Oct. 15)

Turn in only one assignment per group. Please explain clearly what you did to arrive at the answer.

1. a) Using MAPLE, compute the first four derivatives of the function xe^x. Note: e^x is entered as `exp(x)` in MAPLE. Based on the results, what is the n-th derivative of xe^x? To compute the third derivative, you can also enter `diff(x * exp(x), x,x,x)`.

b) Do the same for the function x^2e^x. Compute as many derivatives as you need to observe a pattern, and guess what the n-th derivative of x^2e^x shoud be, for arbitrary n.

2. Notice that the equation $y^2 = x^3 + 3$ has one solution $x = 1, y = 2$. Use the tangent line method in the example 2 above to find two more rational solutions. Leave your answer as a fraction with integer numerator and denominator.

3. Write up your solution to the in-class exercise 3.